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RATIONAL EXPECTATIONS, INVENTORIES AND PRICE FLUCTUATIONS

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ABSTRACT

This paper considers a linear-quadratic stochastic model of a market for a storable commodity in which private agents speculate in inventories subject to storage costs. It is shown that the non-negativity constraint on inventories will be binding. The competitive rational expectations equilibrium is calculated for a special case and it is shown that speculators adopt a reservation price strategy. Furthermore, the equilibrium is shown to be optimal. Hence, conventional arguments for price stabilization using public buffer stocks based on consumer-producer surplus analysis cannot be justified in the context of the type of model considered here.

I. Introduction

Following the work of Waugh [12], Oi [5] and Massel [3], the role of inventories and the welfare implications of various price stabilization schemes have received much attention in the literature. However, most of these studies, with the exception of Turnovsky [11], have proceeded by ruling out private inventories. The typical approach has been to append additive or multiplicative "white noise" disturbances to demand and supply functions. The benefits accruing to consumers and producers from buffer stock policies aimed at fixing the market price are then analyzed.[1] However, buffer stock policies designed to maintain fixed prices are not feasible because the stock of government held inventories will follow a random walk.[2] In the absence of arguments to the effect that the storage technology is available only to the government, a re-examination of feasible price stabilization schemes should be conducted in the framework of a model where private agents are allowed to hold inventories.

Turnovsky's [11] model, which incorporates private storage, is based upon Muth's [4] model of speculative behavior, in which inventories of final goods are held only for speculative gains.

We first show that Muth's linear demand function for speculative inventories can be obtained in a more straightforward way. It is important to note that in Muth's analysis, inventories are not constrained to be non-negative. However, there seems to be no reasonable economic interpretation to the notion of negative inventories in this model. The suggestion that one should interpret negative inventories as backlogs requires that we assume that consumers faced with perfectly competitive firms are willing to pay for a good today which they will receive tomorrow while other consumers, indistinguishable from them, pay the same price and receive the same good today. Such a story is incompatible with specifying demand as a function of the current price only. As it turns out, in the Muth model, purely speculative inventories must be negative for some periods.

As such we re-solve the speculator's problem subject to the nonnegativity constraint and compute the resulting rational expectations competitive equilibrium. This equilibrium has the property that speculators adopt a reservation price policy. In particular, inventories are positive when the market price is below some endogenously determined reservation price. Otherwise, inventories are zero.

Finally, we consider the implications of the model for price stabilization schemes. Because the model is based on explicit microeconomic foundations, the welfare analysis can be carried out in terms of a defensible social welfare function. If the social planner were to maximize the expected value of the discounted sum of consumers' plus producers' surplus, the planner would reproduce the rational expectations competitive equilibrium.[3] Hence, the optimal storage policy is not one aimed at fixing prices, even if such a policy is feasible.

The remainder of this paper is organized as follows. In section II we reproduce the Muth/Turnovsky model assuming speculators maximize expected discounted profits from storage. In section III we impose the nonnegativity constraint and solve for the rational expectations competitive equilibrium. In section IV we consider a simulated example. Finally, in section V we discuss the implications of the model of section III for price stabilization policies.

II. The Model

In this section we reproduce the Muth/Turnovsky model by specifying explicit optimization problems for the producers and the speculators. Then we show that in this model the non-negativity constraint on inventories is necessarily binding. Hence, the model should be re-solved. Let there be N identical producers in a market for a storable commodity.

The production function of the representative producer is [4]

$$(1) \quad Q_t = \min\{\alpha_1 A_{t-1},\ \alpha_2\sqrt{L_{t-1}}\} + \varepsilon_t$$

where

Q_t = aggregate production at time t,

$\alpha_1 > 0, \ \alpha_2 > 0$

A_{t-1} and L_{t-1} are the aggregate quantities of the two inputs at time t - 1, we assume that the input prices are fixed

and ε_t is an aggregate shock to production which has the law of motion

$$(2) \quad \varepsilon_t = F(L)V_t;\ F(L) = \sum_{j=0}^{\infty} F_j L^j\ ,\ F_0 = 1,\ \text{L is the lag operator,}$$

$$\text{and } E(V_t V_s) = \begin{cases} \sigma_v^2 & s = t \\ 0 & s \neq t \end{cases}$$

$$E(V_t) = 0 \text{ for all } t$$

The problem of the representative producer who maximizes the expected value of discounted profits is[5]

$$(3) \quad \underset{\{A_t\}_{t=0}^{\infty}}{\text{Maximize}}\ E_0 \lim_{T\to\infty} \sum_{t=0}^{T} \beta^t \{P_t(\alpha_1 A_{t-1} + \varepsilon_t) - \frac{\delta}{2} A_t^2 - RA_t\}$$

subject to

A_{-1} given,

$\varepsilon_t = F(L)V_t$,

P_t is the market price of the good at time t and is assumed to be a well defined stochastic process,

$0 < \beta < 1$ is a discount factor,

$\delta > 0$ is a constant determined by α_1, α_2 and the given constant price of L_t,

$R > 0$ is the price of A_t .

Let E_t denote the conditional expectation operator defined on information known at time t. The information set at time t, Ω_t, is assumed to be common to all agents in the model, and includes the values of all variables in the model occuring at time t - s, for all $s \geq 0$.

Then the first-order necessary conditions for the producer's problem yields the following stochastic linear supply function:

(4) $$Q(t) = bE_{t-1}(P_t)\ \ C + \varepsilon_t$$

where

$$b = \frac{\beta\alpha_1^2}{2}\ ,\quad C = \frac{\alpha_1 R}{\delta}\ .$$

We now consider the problem of the representative speculator, whose only gains from holding inventories are those that accrue to him by "buying low and selling high."

If I_t = the aggregate stock of inventories held at the end of time t, and $d > 0$ is a cost parameter, then the problem of the speculator who maximizes the expected discounted value of profits from buying and selling inventories of finished goods is

(5) $$\underset{\{I_t\}_{t=0}^{\infty}}{\text{Maximize}}\ E_0 \lim_{T\to\infty} \sum_{t=0}^{T} \beta^t \{P_t(-I_t + I_{t-1}) - \frac{1}{2d} I_t^2\}$$

subject to

I_{-1} given and the law of motion for the stochastic process $\{P_t\}$.

If the nonnegativity constraint is not imposed upon I(t), the first-order necessary condition for problem (5) is

(6) $$I_t = d[\ E_t\ (P_{t+1}) - P_t].$$

Let the market demand curve for final consumption of the good be given by: [6]

(7) $$D_t = A - aP_t\ ,\quad A > 0,\ a > 0\ .$$

To complete the model we impose the market clearing condition:

(8) $$D_t + I_t = Q_t + I_{t-1}\ .$$

Notice that if we set $\beta = 1$. equation (6) corresponds to Muth's inventory demand function and equations (4), (6), (7) and (8) correspond to the model derived by Muth [4] and used by Turnovsky [11]. However, the above derivation does not require the asymmetric assumptions that Muth makes regarding the objective functions of producers and speculators. Whereas in our setup both groups maximize expected discounted profits, in Muth's paper speculators maximize the expected utility from speculative profits. Because of this, in order to derive his inventory demand equation Muth assumes that the conditional variance of prices at time $t + 1$ is independent of the conditional mean of P_{t+1} , and more importantly, that the squareof the expected price change is small relative to variance of the price.

It is important to notice that in problem (5) no nonnegativity constraint on inventories is imposed. We now show that inventories cannot be non-negative for all t in the above model.

Suppose, if possible, that $I_t > 0$ for all t. Then from (6) we require that

$$\beta E_t(P_{t+1}) \geq P_t \quad \text{for all } t. \tag{9}$$

Therefore,

$$\begin{aligned} E_t(P_{t+j}) &= E_t[E_{t+j-1}(P_{t+j})] \geq E_t\left(\frac{1}{\beta} P_{t+j-1}\right) \\ &= \frac{1}{\beta} E_t[E_{t+j-2}(P_{t+j-1})] \quad \frac{1}{\beta} E_t\left(\frac{1}{\beta} P_{t+j-2}\right) = \cdots \\ &= \frac{1}{\beta^j} P_t \quad \text{for all } j > 0. \end{aligned} \tag{10}$$

But this implies that expression (3) for producer's profits are unbounded and the resulting model is not well defined.[7] This result simply says that in a market for a final commodity we do not expect that the relative price will rise at an exponential rate greater than one, if production and demand are stationary. Furthermore, if the real expected rate of return on holding the commodity is negative, no one would like to hold it. Hence, there is no storage of the commodity.

If one imposes the constraint that $I_t \geq 0$ for all t, the relevant first-order conditions for the problem of the speculators are:

$$(6') \qquad I_t = \begin{cases} d[\beta E_t (P_{t+1}) - P_t] & \text{if } \beta E_t(P_{t+1}) \geq P_t \\ 0 & \text{otherwise} \end{cases}$$

III. The Rational Expectations Competitive Equilibrium

Given the impossibility of problem (5) yielding a solution for I_t which is non-negative for all t, we now solve for the rational expectations competitive equilibrium taking the nonnegativity constraint on I_t into account. We assume that the shock to production, ε_t, has the following probability distribution:

$$(11) \qquad \varepsilon_t = \begin{cases} \varepsilon_1 & \text{with probability } \Pi_1 \text{ for all t} \\ \varepsilon_2 & \text{with probability } \Pi_2 \text{ for all t} \end{cases}$$

$$\Pi_1 + \Pi_2 = 1, \ \varepsilon_1 > \varepsilon_2, \ \Pi_1\varepsilon_1 + \Pi_2\varepsilon_2 = 0 .$$

It is easy to see that for the model under consideration, if there were no inventories, the sequence of equilibrium prices would be i.i.d. However, the existence of durable goods and stocks of inventories will in general introduce serial correlation into the price process. Furthermore, it seems reasonable to conjecture that given the above specification for the ε_t process, if the current market price is "high," then expected price next period will almost surely be lower and inventories would not be held. The above considerations lead to the idea of an inventory holding reservation price, $\overset{*}{P}$, such that $I_t > 0$ if $P_t < \overset{*}{P}$ and $I_t = 0$ if $P_t > \overset{*}{P}$. As such we begin with the following guess regarding the equilibrium price process.

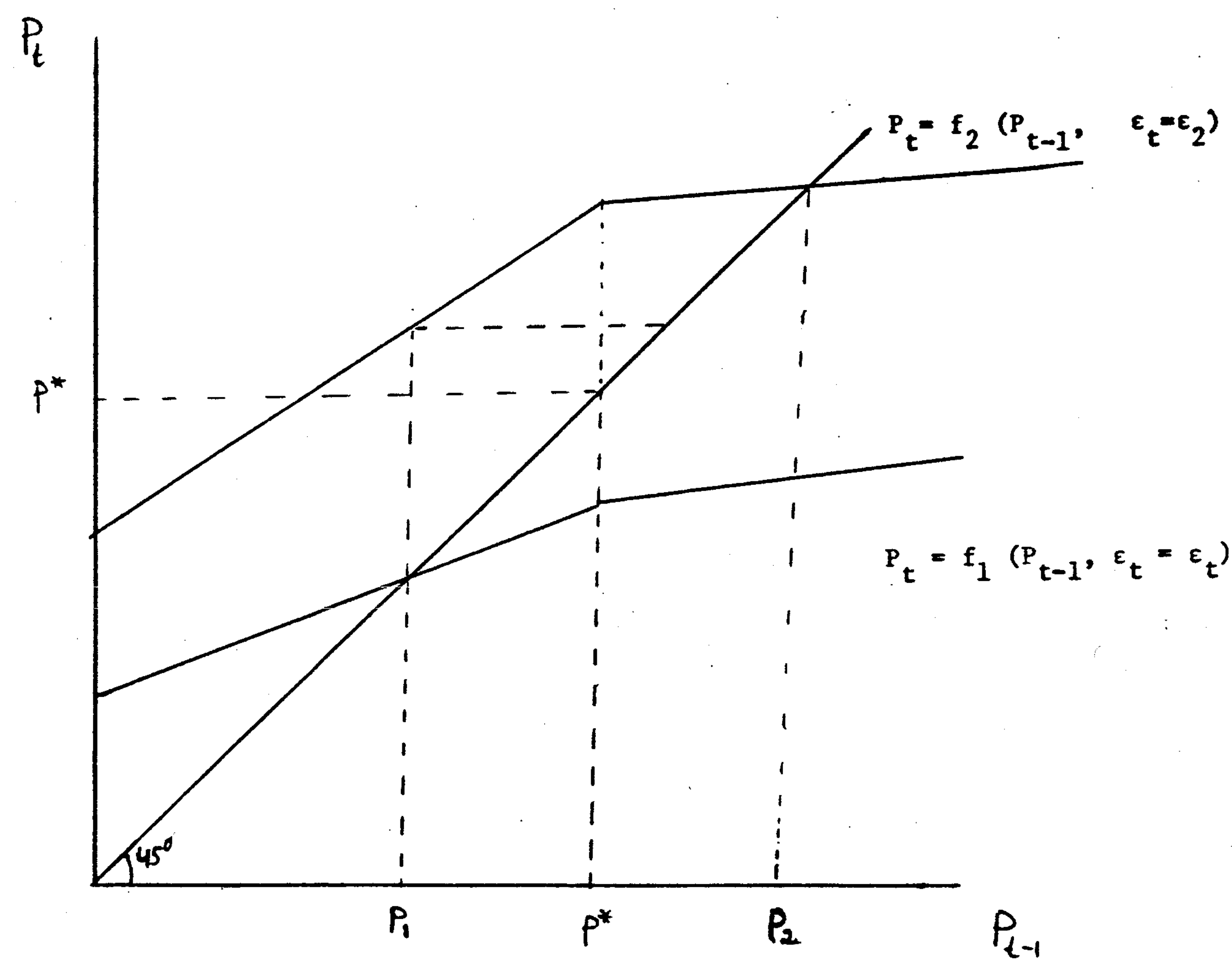

Figure 1

Notice that in the above figure, the functions f_1 and f_2 are parameterized by the realization of the $\varepsilon(t)$ process and have a kink at $\overset{*}{P}$ which implies equations (12) and (13).[8]

$$P_t = \lambda_1(\varepsilon_t)P_{t-1} + \mu_1(\varepsilon_t) \ , \quad P_1 \leq P_{t-1} \leq \overset{*}{P} \tag{12}$$

$$P_t = \lambda_2(\varepsilon_t)P_{t-1} + \mu_2(\varepsilon_t) \ , \quad \overset{*}{P} \leq P_{t-1} \leq P_2 \ . \tag{13}$$

As is evident from Figure 1, we assume that

(14) $\lambda_1(\varepsilon_2)P_1 + \mu_1(\varepsilon_2) > \overset{*}{P}$ which implies that if $P_1 \leq P_{t-1} \leq \overset{*}{P}$ and $\varepsilon_t = \varepsilon_2$ then $\overset{*}{P} \leq P_t \leq P_2$.

(15) $\lambda_2(\varepsilon_1)P_2 + \mu_2(\varepsilon_1) < \overset{*}{P}$ which implies that if $\overset{*}{P} \leq P_{t-1} \leq P_2$ and $\varepsilon_t = \varepsilon_1$, then $P_1 \leq P_t \leq \overset{*}{P}$.[9]

The inventory holding reservation price $\overset{*}{P}$ is such that

$$P_1 \leq P_t \leq \overset{*}{P} \Rightarrow I_t \geq 0 \tag{16}$$

$$\overset{*}{P} \leq P_t \leq P_2 \Rightarrow I_t = 0 \ .$$

Given the above specification there exist four possible cases for the market clearing condition (8) corresponding to the two ranges for P_{t-1} and the two realizations for ε_t.

Case 1

$P_1 \leq P_{t-1} \leq \overset{*}{P}$, $\varepsilon_t = \varepsilon_1$ which implies $P_1 \leq P_t \leq \overset{*}{P}$ and $I_{t-1} \geq 0$ and $I_t \geq 0$.

By taking conditional expectations on the relevant price processes and substituting into the supply and inventory demand functions, market clearing requires that

$$A - aP_t + d[\beta(\bar{\lambda}_1 P_t + \bar{\mu}_1) - P_t] = b[\bar{\lambda}_1 P_{t-1} + \bar{\mu}_1] - C + \varepsilon_1$$
$$+ d[\beta(\bar{\lambda}_1 P_{t-1} + \bar{\mu}_1) - P_{t-1}]$$

where the "-" symbol stands for the expected value. Substituting (12) with $\varepsilon_t = \varepsilon_1$ into the above expression and equating the coefficients of P_{t-1} on both sides, as well as the constant terms, we obtain

(17) $$\lambda_1(\varepsilon_1)[a + d(1 - \beta\bar{\lambda}_1)] = d(1 - \beta\bar{\lambda}_1) - b\bar{\lambda}_1$$

(18) $$\mu_1(\varepsilon_1)[a + d(1 - \beta\bar{\lambda}_1)] = A + C - \varepsilon_1 - b\bar{\mu}_1 \ .$$

Case 2

$P_1 \leq P_{t-1} \leq \overset{*}{P}$, $\varepsilon_t = \varepsilon_2$ which implies that $\overset{*}{P} \leq P_t \leq P_2$, and $I_{t-1} \geq 0$ and $I_t = 0$.

Proceeding as above, we obtain

(19) $$a\lambda_1(\varepsilon_2) = d(1 - \beta\bar{\lambda}_1) - b\bar{\lambda}_1$$

(20) $$a\mu_1(\varepsilon_2) = A + C - \varepsilon_2 - (b + \beta d)\bar{\mu}_1 \ .$$

Case 3

$\overset{*}{P} \leq P_{t-1} \leq P_2$, $\varepsilon_t = \varepsilon_1$ which implies that $P_1 \leq P_t \leq \overset{*}{P}$ and $I_{t-1} = 0$ and $I_t \geq 0$.

Proceeding as above, we obtain

(21) $$\lambda_2(\varepsilon_1)[a + d(1 - \beta\bar{\lambda}_1)] = -b\bar{\lambda}_2$$

(22) $$\mu_2(\varepsilon_1)[a + d(1 - \beta\bar{\lambda}_1)] = A + C - \varepsilon_1 - b\bar{\mu}_2 + \beta d\bar{\mu}_1 .$$

Case 4

$\overset{*}{P} \leq P_{t-1} \leq P_2$, $\varepsilon_t = \varepsilon_2$ which implies that $\overset{*}{P} \leq P_t \leq P_2$ and $I_{t-1} = I_t = 0$.

Hence,

(23) $$a\lambda_2(\varepsilon_2) = -b\bar{\lambda}_2$$

(24) $$a\mu_2(\varepsilon_2) = A + C - \varepsilon_2 - b\bar{\mu}_2 .$$

Equations (17) - (24) can be solved for the eight unknowns $\lambda_1(\varepsilon_t)$, $\lambda_2(\varepsilon_t)$, $\mu_1(\varepsilon_t)$, $\mu_2(\varepsilon_t)$, $\varepsilon_t = \varepsilon_1, \varepsilon_2$, as follows.

Solve (17) and (19) for $\lambda_1(\varepsilon_1)$ and $\lambda_1(\varepsilon_2)$ in terms of $\bar{\lambda}_1$. Then take expectations $\Pi_1\lambda_1(\varepsilon_1) + \Pi_2\lambda_1(\varepsilon_2) = \bar{\lambda}_1$, to get:

(25) $$\bar{\lambda}_1 = [d(1 - \beta\bar{\lambda}_1) - b\bar{\lambda}_1][\frac{\Pi_1}{a + d(1 - \beta\bar{\lambda}_1)} + \frac{\Pi_2}{a}]$$

which is a quadratic expression in $\bar{\lambda}_1$. Letting $X = 1 - \beta\bar{\lambda}_1$ we may write (25) as:

(26) $$f(X) = X^2[d + \Pi_2 \frac{d}{a} (b + \beta d)] + X[a + b - \Pi_2 \frac{db}{a} - d(1 - \beta)] - (a + b) = 0 .$$

Since $f(0) < 0$ and $f(1) > 0$ there exists a root X such that $0 < X_1 < 1$. Hence, there exists a solution to (25) with $0 < \bar{\lambda}_1 < 1/\beta$. Now since $\bar{\lambda}_1 > 0$, either $\lambda_1(\varepsilon_1) > 0$ or $\lambda_1(\varepsilon_2) > 0$ (or both). Comparing (17) and (19) and noting that $\bar{\lambda}_1\beta < 1$, we see that $\lambda_1(\varepsilon_1) > 0$ if and only if $\lambda_1(\varepsilon_2) > 0$. Hence

(27) $$\begin{cases} 0 < \lambda_1(\varepsilon_1) < 1 \\ 0 < \lambda_1(\varepsilon_1) < \bar{\lambda}_1 < \lambda_1(\varepsilon_2) . \end{cases}$$

Proceeding in a similar way we solve (18) and (20) for $\mu_1(\varepsilon_1)$ and $\mu_1(\varepsilon_2)$. By taking expectations, $\Pi_1\mu_1(\varepsilon_1) + \Pi_2\mu_1(\varepsilon_2) = \bar{\mu}_1$, we obtain:

$$\bar{\mu}_1 = \Pi_1\left[\frac{A + C - \varepsilon_1 - b\bar{\mu}_1}{a + d(1 - \beta\bar{\lambda}_1)}\right] + \Pi_2\frac{[A + C - \varepsilon_2 - (b + \beta d)\bar{\mu}_1]}{a} . \tag{28}$$

By using the solution for $\bar{\lambda}_1$ obtained from (25) we can solve (28) for $\bar{\mu}_1$, after which we solve for $\mu_1(\varepsilon_1)$ and $\mu_1(\varepsilon_2)$ from (18) and (20). It can easily be shown that $\bar{\mu}_1 > 0$.

Following the same procedure, we can solve (21) and (23) for $\lambda_2(\varepsilon_1)$ and $\lambda_2(\varepsilon_2)$ to obtain $\lambda_2(\varepsilon_1) = \lambda_2(\varepsilon_2) = \bar{\lambda}_2 = 0$. Notice that this indicates that in the absence of inventories prices will be serially uncorrelated. In a similar way we can solve (22) and (24) for $\mu_2(\varepsilon_1)$, $\mu_2(\varepsilon_2)$ and $\bar{\mu}_2 = \Pi_1\mu_2(\varepsilon_1) + \Pi_2\mu_2(\varepsilon_2)$.

Because the suggested solutions (12) and (13) should be continuous at $\overset{*}{P}$ for each realization of ε_t, we require that

$$\begin{cases} \lambda_1(\varepsilon_1)\overset{*}{P} + \mu_1(\varepsilon_1) = \mu_2(\varepsilon_1) \\ \lambda_1(\varepsilon_2)\overset{*}{P} + \mu_1(\varepsilon_2) = \mu_2(\varepsilon_2) \end{cases} \tag{29}$$

or

$$\overset{*}{P} = \frac{\mu_2(\varepsilon_1) - \mu_1(\varepsilon_1)}{\lambda_1(\varepsilon_1)} = \frac{\mu_2(\varepsilon_2) - \mu_1(\varepsilon_2)}{\lambda_1(\varepsilon_2)} .$$

The latter equality may be verified directly by use of earlier expressions derived. Hence, $\overset{*}{P}$ is uniquely defined.

By multiplying the first equation in (29) by Π_1 and the second equation in (29) by Π_2 and adding we see that

$$\overset{*}{P} = \frac{\bar{\mu}_2 - \bar{\mu}_1}{\bar{\lambda}_1}$$

and by using (18) for $\mu_1(\varepsilon_1)$, (17) for $\lambda_1(\varepsilon_1)$ and (22) for $\mu_2(\varepsilon_1)$, we obtain:

$$\text{(30)} \qquad \overset{*}{P} = \frac{b(\bar{\mu}_2 - \bar{\mu}_1) - \beta d\bar{\mu}_1}{\bar{\lambda}_1(b + \beta d) - d} = \frac{\bar{\mu}_2 - \bar{\mu}_1}{\bar{\lambda}_1} .$$

Hence,

$$\text{(31)} \qquad \overset{*}{P} = \frac{\beta\bar{\mu}_1}{1 - \beta\bar{\lambda}_1} > 0$$

since $\bar{\mu}_1 > 0$, which from (30) implies that $\bar{\mu}_2 > \bar{\mu}_1$.

We now verify our guess (16) that

$$P_1 \leq P_t \leq \overset{*}{P} \Rightarrow I_t \geq 0$$

$$\overset{*}{P} \leq P_t \leq P_2 \Rightarrow I_t = 0 .$$

For $P_t \leq \overset{*}{P}$,

$$\beta E_t(P_{t+1}) - P_t = \beta[\bar{\lambda}_1 P_t + \bar{\mu}_1] - P_t = \beta\bar{\mu}_1 - P_t(1 - \beta\bar{\lambda}_1) \geq 0 .$$

Hence, $I_t \geq 0$ for $P_t \leq \overset{*}{P}$.

For $P_t \geq \overset{*}{P}$,

$$\beta E_t(P_{t+1}) - P_t = \beta\bar{\mu}_2 - P_t \leq 0$$

since from (30) and (31) $\overset{*}{P} = \beta\bar{\mu}_2$. Hence, $I_t = 0$ for $P_t \geq \overset{*}{P}$.

Figure 2 depicts the resulting equilibrium price process. Notice that

$$P_1 = \frac{\mu_1(\varepsilon_1)}{1 - \lambda_1(\varepsilon_1)}$$

$$P_2 = \mu_2(\varepsilon_2) .$$

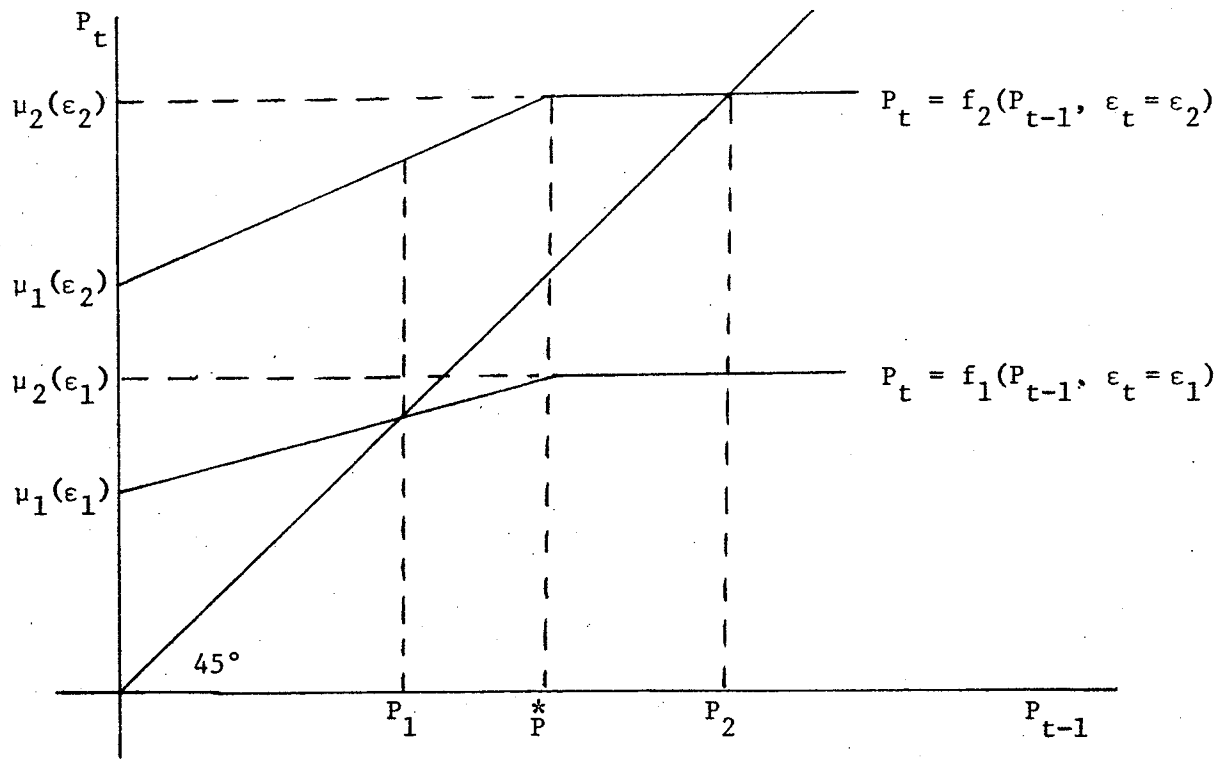

Figure 2

While condition (15) is automatically satisfied by our solution, condition (14) and the requirement that $0 < P_1 < \overset{*}{P} < P_2$ impose restrictions on the parameters of the model.[10]

Distribution of Inventories

Because P_t is serially correlated we expect I_t to be serially correlated. This turns out to be the case and the behavior of inventories can be represented as in Figure 3.

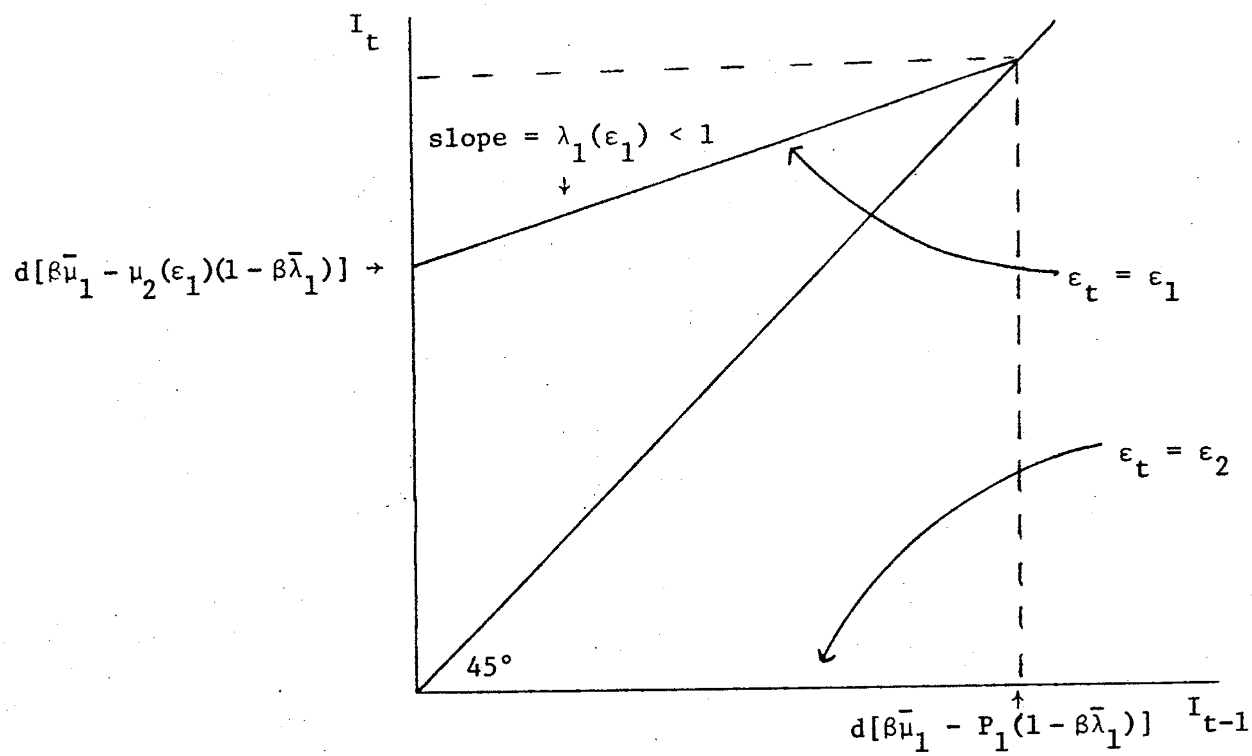

Figure 3

Let $d[\beta\bar{\mu}_1 - P_1(1 - \beta\bar{\lambda}_1)] = \overset{*}{I}$ and $\lambda_1(\varepsilon_1) = \alpha$. Then inventories will take on the values $\{0, (1 - \alpha)\overset{*}{I}, (1 - \alpha^2)\overset{*}{I}, (1 - \alpha^N)\overset{*}{I}, \ldots\}$ with the correspondi probabilities $\{\Pi_2, \Pi_2\Pi_1, \Pi_2\Pi_1^2, \ldots, \Pi_2\Pi_1^N, \ldots\}$. It is easy to show that the unconditional expected value of inventories is equal to

$$E[I_t] = \frac{\Pi_1(1 - \alpha)\overset{*}{I}}{1 - \Pi_1\alpha}\ .$$

Furthermore, the probability of observing zero as opposed to positive inventories is Π_2, the probability of a negative shock to output. Hence, if Π_2 is small, inventories will be positive most of the time.

An interesting feature of the solution, as depicted in Figure 3, is that starting from zero inventories, a succession of "good" shocks ($\varepsilon_t = \varepsilon_1$) results in a build up of inventories (at a decreasing rate) and then a complete decumulation of stocks when a "bad" shock is realized.[11]

It can also be shown that a larger value of d, i.e., a lower cost of holding inventories, results in a higher expected value of inventories.

By using (31) we may write the optimal inventory decision rule as

$$(32) \qquad I_t = \begin{cases} d(1 - \beta\bar{\lambda}_1)[\overset{*}{P} - P_t] & \text{for } P_t \leq \overset{*}{P} \\ 0 & \text{for } P_t > \overset{*}{P} \end{cases}$$

(32) emphasizes the role of the reservation price $\overset{*}{P}$ and demonstrates that the level of inventories, when positive, is directly proportional to the difference between $\overset{*}{P}$ and P_t.[12]

IV. An Example

We now consider a numerical example and examine the characteristics of the simulated time series emerging from the rational expectations competitive equilibrium in order to demonstrate the non-vacuousness of the solution proposed in section III. We then compare the model in which the nonnegativity constraint is imposed with the model in which it is not imposed and to the case where private storage is outlawed. Finally, the variance of prices in the three models is discussed.

Let

$$\beta = .9,\ d = 10.0,\ b = 5.0,\ A = 12.0,\ a = 5.0,\ C = 2.0,$$

$$\varepsilon_1 = 1.0,\ \Pi_1 = .8,\ \varepsilon_2 = -4.0,\ \Pi_2 = .2\ .$$

Case 1: For the model in Section III we obtain the following solution:

$$P_1 = 1.209, \quad \overset{*}{P} = 1.28, \quad P_2 = 2.177, \quad \lambda_1(\varepsilon_1) = .35.233,$$

$$\lambda_1(\varepsilon_2) = .778,, \; \mu_1(\varepsilon_1) = .784 \;, \; \mu_1(\varepsilon_2) = 1.182 \;, \; \mu_2(\varepsilon_1) = 1.234 \;,$$

$$\mu_2(\varepsilon_2) = 2.177, \quad \bar{\lambda}_1 = .437, \quad \bar{\mu}_1 = .864, \quad \bar{\mu}_2 = 1.423$$

and the equilibrium sequence of prices is determined by

$$P_t = \begin{cases} .351 \; P_{t-1} + .784 & \text{for } P_1 \leq P_{t-1} \leq \overset{*}{P}, \; \varepsilon_t = \varepsilon_1 \\ .778 \; P_{t-1} + 1.182 & \text{for } P_1 \leq P_{t-1} \leq \overset{*}{P}, \; \varepsilon_t = \varepsilon_2 \end{cases}$$

$$P_t = \begin{cases} 1.234 & \text{for } \overset{*}{P} \leq P_{t-1} \leq P_2, \; \varepsilon_t = \varepsilon_1 \\ 2.177 & \text{for } \overset{*}{P} \leq P_{t-1} \leq P_2, \; \varepsilon_t = \varepsilon_2 \end{cases}$$

$$I_t = \begin{cases} -6.071 \; P_t + 7.773 & P_t \leq 1.28 \\ 0 & P_t \geq 1.28 \end{cases}$$

$$Q_t = 5.0 \; E_{t-1}(P_t) - 2.0 + \varepsilon(t) \; .$$

Case 2: The solution to the model if the non-negativity constraint on inventories is not imposed is:

$$P_t = .850 + .392 \; P_{t-1} - .087 \; \varepsilon_t$$

$$I_t = -6.466 \; P_t + 7.652$$

$$D_t = 12.0 - 5.0 \; P_t$$

$$Q_t = 1.826 - 1.963 \; P_{t-1} + \varepsilon_t$$

Case 3: The solution to the model if inventories are outlawed is:

$$P_t = 1.4 - 0.2 \; \varepsilon_t$$

The variance of prices for each of the above three cases are as follows:

Case 1: $\sigma_p^2 = 0.096$

Case 2: $\sigma_p^2 = 0.036$

Case 3: $\sigma_p^2 = 0.16$

In cases 2 and 3 the variances shown are population variances, while in case 1 the variance shown was calculated from a simulated time series on prices with one hundred observations.

It should not come as a surprise that the variance of prices is largest when inventories are outlawed (0.16), followed by the case in which inventories are allowed but constrained to be non-negative (0.096) and that the variance of prices is smallest when the non-negativity constraint on inventories is not imposed (0.036). The effect of inventories is clearly toward a reduction of price variance.

It is also interesting to compare the solutions for cases 1 and 2. The solution for inventories in case 1 (when they are positive) is quite close to the solution for inventories in case 2. However, the solution in the two cases for the equilibrium price process is quite different. This results in inventories having a negative mean for case 2, whereas in case 1 inventories are never negative by construction.

V. Welfare

We now consider the social planning problem which consists of maximizing the expected value of the sum of discounted consumer plus producer surplus. We show that the social planner would choose to reproduce the rational expectations competitive equilibrium. Furthermore, aside from questions of feasibility, any policy that attempts to fix the market price does not maximize the market surplus.

The social planner problem for the model of section III is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \{\frac{A}{a} D_t - \frac{1}{2a} D_t^2 - \frac{\delta}{2} A_t^2 - RA_t - \frac{1}{2d} I_t^2 - \lambda_t [D_t + I_t - I_{t-1} - \alpha_1 A_{t-1} - \varepsilon_t]\} \tag{33}$$

by choice of

$$(D_t, A_t, I_t, \lambda_t) \geq 0, \ t = 0, 1, 2, \ldots$$

subject to A_{-1} and I_{-1} given.

Notice that

$\frac{A}{a} D_t - \frac{1}{2a} D_t^2$ = area under the demand curve from 0 to D_t.

$RA_t + \frac{\delta}{2} A_t^2$ = costs of production,

$\frac{1}{2d} I_t^2$ = costs of holding inventories.

$$D_t + I_t \leq Q_t + I_{t-1} = \alpha_1 A_{t-1} + \varepsilon_t + I_{t-1} \tag{34}$$

is the material balance constraint. $\{\lambda_t\}$ = Lagrange multipliers associated with (34).

The first-order necessary conditions for problem (33) are

$$\frac{A}{a} - \frac{D_t}{a} - \lambda_t = 0 \tag{35}$$

$$A_{t-1} = \frac{\beta\alpha_1}{\delta} E_{t-1}(\lambda_t) - \frac{R}{\delta} \tag{36}$$

$$I_t = \begin{cases} d[\beta E_t(\lambda_{t+1}) - \lambda_t] & \text{if } \beta E_t(\lambda_{t+1}) - \lambda_t \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{37}$$

$$(38) \qquad D_t + I_t - I_{t-1} - \alpha_1 A_{t-1} - \varepsilon_t = 0 .$$

It is clear that the set of equations (35) - (38) is equivalent to the set of equations (4), (6'), (7) and (8) once we set $\lambda_t = P_t$ and $A_{t-1} = (Q_t - \varepsilon_t)/\alpha_1$. Hence, any solution to the rational expectations competitive equilibrium model of section III is also a solution to the social planner problem. Furthermore, because the objective function in (33) is strictly concave, problem (33) has a unique solution. Hence, there exists no other allocation that can increase both consumers' and producers' surplus, over that which is attained by the rational expectations competitive equilibrium.

The immediate implications of this result may be summarized as follows:

(a) The fact that some studies[13] show that buffer stock policies which attempt to fix the market price may increase the market surplus is due to the fact that the solutions in those papers do not correspond to the rational expectations equilibrium. It should not come as a surprise that if economic agents do not optimize then there exist welfare improving government interventions. However, the best policy is not one which is aimed at minimizing price variance.

(b) The "social planner" problem emphasizes that inventories increase welfare by "smoothing" consumption over time. While this may result in a reduction of price fluctuations, the welfare gains arise from the smoothing of quantities, not of prices.

(c) The qualitative solution of the model is independent of the market structure imposed, namely perfect competition. For example, one can solve for the case of a monopolist producer-speculator by solving (33) with the term $(-\frac{1}{2a} D_t^2)$ replaced by $(-\frac{1}{a} D_t^2)$.

(d) The qualitative nature of the solution with regard to inventories is unchanged if the uncertainty in the model enters via an additive shock to demand as opposed to supply.

(e) In the linear-quadratic rational expectations framework adopted here, it makes no difference whether or not there exists a futures market. Turnovsky's [11] comparison of the two models in sections two and three of his paper depends exclusively on the fact that in section two, expectations are not rational, and not on the presence or absence of futures markets.

VI. Conclusion

It is shown that in a simple linear model in which inventories are held for speculative purposes only, inventories must sometimes be zero. One should note that this is not quite equivalent to the existence of stockouts in the conventional sense. Since ours is a competitive equilibrium model, consumers are able to obtain, at the prevailing market price, the amount that they desire to purchase. There are no disappointed consumers.

It seems to us that in the framework of quadratic objective functions, linear constraints and a stationary price process it is almost impossible to have speculative inventories positive for all periods. The resulting "corner" as well as the dynamics inherent in a model with a storable good imply a nontrivial solution to the market equilibrium. We believe that the existence of a reservation price strategy for speculator depends only on the i.i.d. nature of the shocks to the system. We calculate the solution for the case when the random shocks may be characterized by a two point i.i.d. discrete probability distribution. The reservation price character of the optimal decision rule for inventories appears similar to buffer stock policies but is clearly not motivated by any attempt to fix prices. The solution to the model with the nonnegativity constraint imposed is contrasted with the solution when that constraint is neglected.

The welfare implications of price stabilization polices are discussed within the context of the partial equilibrium model presented here. It is shown that the uniquely optimal policy of the social planner is to reproduce the allocations resulting from the rational expectations competitive equilibrium. It seems to us that arguments for price stabilization cannot be justified within the class of models considered in this paper.

FOOTNOTES

[1] For a brief survey, see Wright [13]. A more detailed survey is contained in Turnovsky [10].

[2] See Townsend [8] and Wright [13].

[3] Eichenlaum [1] derives a similar result in a different context. For a more general discussion of the relationship between social planning problems and rational expectations competitive equilibria, see Lucas and Prescott [2].

[4] This production function was chosen in order to obtain a linear supply function with an additive shock. Notice that if the average product is made random, supply will have a multiplicative shock.

[5] The following (sufficient) conditions for problem (3) to be well defined are imposed: There exist

$$K > 0 \text{ and } 0 \leq X < 1/\sqrt{\beta}$$

such that

$$E_t(P_{t+j}) \leq KX^{t+j} \text{ for all } t \text{ and } j > 0$$

We also assume that $\alpha E_{t-1}(P_t) > R$ for all t.

[6] We assume that A, a, C, ε_t and b are such that for the deterministic case, with $I_t = 0$ for all t, the equilibrium price and quantity are positive.

[7] An alternative approach is as follows. Sargent [7, pp. 270-271] solves this model for the case where $\beta = 1$. Following his exposition but with $0 < \beta < 1$, one can show that the equilibrium price process is covariance stationary with positive mean. It can then be shown that the equilibrium inventories process is also covariance stationary but with a negative mean. Hence, inventories cannot always be positive. Similarly for $\beta = 1$, the mean is zero and inventories cannot always be non-negative.

[8] Notice that Figure 1 implies that the equilibrium price will remain in the interval $[P_1, P_2]$ with probability one. Hence, we need only consider the range $[P_1, P_2]$ for P_{t-1}.

[9]Assumptions (14) and (15) clearly impose restrictions on the values of the parameters of the model.

[10]Uniqueness will be proved in section V.

[11]Notice that inventories are bounded from above by $\overset{*}{I}$.

[12]Samuelson [6] considers a similar model of speculative behavior where inventories are subject to proportional 'shrinkage' but there are no quadratic holding costs as in our case. He also takes production as a given exogenous stochastic process independent over time. His result regarding the existence of a reservation price is similar to ours.

[13]See Turnovsky [9, 10, 11].

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