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ECONOMICS OF COMMITTEES

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ECONOMICS OF COMMITTEES

This dangerous fallacy I shall now illumine:
To committees, nothing alien is human

—Ogden Nash

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ABSTRACT

An objective of this paper is to analyze the economic performance of committees; in particular, the optimal degree of consensus to be required for committee's decision making (concerning acceptance or rejection of projects), and the optimal size of committee. Our focus is on two basic trade-offs in organizational decision making: between the (Type-I) errors of rejecting good projects and the (Type-II) errors of accepting bad projects; and between gains from a more extensive evaluation of projects and the resources spent on evaluation. We provide a general characterization of the optimum, and use this characterization to derive a number of qualitative results. For instance, if the two types of individuals' errors of judgment are equal, then the marginal majority rule is optimal (that is, for an increment in the committee size by 2, the optimal consensus increases by 1). If, in addition, the losses from the two types of errors are equal, then the majority rule is optimal.

The paper also analyzes hierarchies, polyarchies, and more complex forms of organization; and derives, for instance, simple interpretations of the optimal number of levels within a hierarchy, and units within a polyarchy. Among other questions which we address are, whether perfection in organizational decision making is feasible, and whether it is economically desirable.
ECONOMICS OF COMMITTEES

Raj Kumar Sah and Joseph E. Stiglitz

Committees represent a widespread form of modern decision making. This is because 'to err is human': on questions of importance, we are often reluctant to delegate the decision making authority to any single individual. There is an implicit belief that the wisdom of a committee is greater than that of any single member, that collective decision making avoids some of the worst errors that might otherwise occur. But there is also a widespread belief that 'to consign a matter to a committee is to consign it to death': it is difficult to get committees to do anything useful. This paper is an attempt to formalize some of these intuitive notions. We construct a model which enables us to analyze the consequences for decision making of committees of different sizes operating under different rules. We show, for instance, that there is some truth in the conventional wisdom that while committees requiring a high degree of consensus do avoid many of the errors of accepting bad projects (or ideas, or people) that an individual would have made, they also reject many good projects which an individual would not have rejected.

Economics is concerned with trade-offs, and the present paper is concerned with the trade-offs involved in the decision making of committees and other organizational forms. There are two trade-offs upon which we focus: first, between the errors of rejecting good projects (Type-I errors) and the errors of accepting bad projects (Type-II
errors); and, second, between the gains from a more extensive evaluation of projects, and the extra resources spent on evaluating projects. For instance, by increasing the size of the consensus required for project approval in a committee of a fixed size, one can decrease the Type-II errors, but only at the expense of increasing Type-I errors. Similarly, by increasing the size of the committee (and changing the decision rule in an optimal way corresponding to the enlarged committee), one may increase the mean quality of projects adopted, but one also increases evaluation costs.

We provide a characterization of the optimal rules for adoption of projects, and of the optimal committee size.\(^1\) As one would expect, the optimal decision rules depend on what kinds of errors individuals make in their evaluation of projects, and on how good or bad is the project portfolio from which they choose. This characterization enables us to delineate intuitive conditions under which majority rule (as required by many committees) is optimal, and under which the optimal decision rule entails more (or less) consensus than that in the majority rule. We show how changes in exogenous variables affect the optimal decision rule. For instance, the greater the relative loss from Type-I errors, the lower the optimal degree of consensus; and the greater the relative loss from Type-II errors, the higher the optimal degree of consensus.

Another objective of this paper is to study certain aspects of hierarchical (centralized) organizations, in which a project is undertaken only if it is approved by the many successive layers of the hierarchy; and of polyarchic (decentralized) organizations, in which any one of the many units can undertake a project, independently of others. (In our 1984 paper, we have contrasted the performance of a hierarchy consisting of two
levels with a polyarchy consisting of two firms).

Clearly, there are some parallels between an $n$ member committee, in which unanimity is required, and an $n$ level hierarchy; and between an $n$ member committee in which one individual's approval is sufficient for project adoption, and an $n$ unit polyarchy. But, as we shall see, there are also some important differences among these organizational forms; particularly concerning the sequence of decision making, and the corresponding evaluation costs. We provide here an analysis of the optimal number of levels in a hierarchy, and the optimal number of units in a polyarchy. We then extend our analysis to more complex organizations which are composed of hierarchies, polyarchies, and committees. Among the questions we address are, whether perfection in organizational decision making is feasible, and whether it is economically desirable.

The paper also provides an insight on why there is such a widespread sense of powerlessness in modern societies, even among individuals who occupy seemingly important decision making positions. One interpretation of this phenomenon is that an individual feels powerless if the collective decision is contrary to his judgment; for example if a project is accepted (rejected) when this individual disapproves (approves) of the project. The theory which we present suggests that when decision making is well organized, the nature of human fallibility is recognized and, thus, this form of powerlessness is an essential counterpart of economic decision making.

This paper is not closely related to the standard literature on voting rules in the theory of social choice. There, the emphasis has been on identifying rules to 'aggregate' different preferences of individuals, which satisfy certain desideratum. Here, values (objectives) of the
members of the organization are the same, but their judgments differ
(possibly because of the incompleteness and the differences in
information). Though, in an abstract sense, these differences in
judgments can be represented as differences in preferences, the particular
structures of organizations within which we study the problem of decision
making allow us to obtain many qualitative insights.

It appears to us that the situations which we describe here, where the
central differences among individuals are in their judgments, not in their
values, arise frequently in economic contexts. Even in political
contexts, such as national elections, the extent to which collective
choices are made on the basis of judgments (concerning the competence of
candidates) rather than on values seems, at best, a moot question.

The paper is organized as follows. The basic model for analyzing the
central trade-offs is presented in section I. We use this model, in
sections II and III, to characterize and interpret the optimal decision
rules for committees' project acceptance, and the optimal committee size.
Section IV contains an analysis of decision making in hierarchies,
polyarchies, and more complex forms of organizations. Concluding remarks
are presented at the end.

I. THE MODEL

There are \( n \) members in a committee, whose task is to accept or
reject projects. The size of (minimum required) consensus for accepting a
project is represented by \( k \). That is, a project is accepted only if \( k \)
or more members accept it; otherwise it is rejected. \( n \geq k \geq 0 \), and
\( n > 1 \). There are two kinds of projects, good and bad, with respective
(net expected) profits $x_1$ and $-x_2$, where $x_1$ and $x_2$ are positive. $a$ is the proportion of good projects: $1 > a > 0$. An individual has some, but not perfect, ability to distinguish between good and bad projects. Thus, if $p_1$ and $p_2$ represents, respectively, the probabilities that an individual committee member accepts a good and a bad project, then $1 > p_1 > p_2 > 0$. One can interpret $1 - p_1$ and $p_2$ as the Type-I and Type-II errors entailed in an individual's decision making. As we shall see, this interpretation is highly useful in understanding our results.

We assume at present that all committee members are homogeneous in their decision making abilities, and they judge a project simultaneously (thus, the committee is not composed of subcommittees). Later, we examine certain decision structures in which the evaluation of projects is sequential. The probability that a project of type $i$ is accepted by the committee is

$$h_i = h(k, n, p_i) = \sum_{j=k}^{n} \binom{n}{j} p_i^j (1 - p_i)^{n-j},$$

where $i = 1$ and 2, and $1 - h_1$ and $h_2$ can be interpreted as the Type-I and Type-II errors entailed in a committee's decision making.

Three intuitive properties of the above expression which we shall use later are as follows. First, a committee of a given size is less likely to accept a project (good or bad) if it requires a larger consensus. This can be seen directly from (1) which yields

$$h(k + 1, n) - h(k, n) = \binom{n}{k} p^k (1 - p)^{n-k} < 0.$$  

Thus, $h$ is decreasing in $k$. Second, for a given consensus
requirement, a larger committee is more likely to accept a project. Specifically,

\[ h(k, n + 1) - h(k, n) = \binom{n}{k-1} p^k (1-p)^{n-k+1} > 0, \]

which shows that \( h \) is increasing in \( n \). \(^3\)

Both of these properties have simple interpretations. A larger required consensus or a smaller committee size implies that every project is being subjected to a tighter scrutiny; as a result, the probability that a project is accepted by the committee is lower. Since this is true for good as well as bad projects, it has an immediate implication: A larger required consensus or a smaller committee size leads to a greater incidence of Type-I errors and a smaller incidence of Type-II errors in committee's decisions.

A third, obvious, property of expression (1) is that a project is more likely to be accepted by a committee if the probability of its acceptance by individuals is higher. That is \(^4\)

\[ \frac{\partial h}{\partial p} > 0. \]

The (expected) profit of a committee is given by

\[ Y = a_1 h_1 - a_2 h_2 - C(n) \]

where \( a_1 = a z_1, \ a_2 = (1 - a) z_2 \). \( C(n) \) is the evaluation cost for a single project. \( C \) depends on the size of the committee, \( n \), and it is increasing and convex in \( n \). In (5) and in the rest of the paper, we suppress the number of projects in the project portfolio faced by the committee.
II. DECISION RULES FOR COMMITTEES OF FIXED SIZE

Recall that a larger consensus requirement has the advantage that fewer bad projects are accepted, but also the disadvantage that a larger number of good projects are rejected. Therefore, as one would expect, the optimal decision rule (consensus) depends on how good, or bad, the project portfolio is, and what is the nature of individuals' errors. Our objective in this section is to identify some of the properties of the optimal decision rule. In particular, we delineate sufficient conditions under which the majority rule is optimal, and under which the optimal decision rule entails a larger, or a smaller, consensus than that in the majority rule. Also, we ascertain the effects of exogenous parameters on the optimal consensus.

A. Optimum

In Appendix I(a), we show that: A committee's profit is single peaked in $k$, and that the optimal $k$ is characterized by the expressions (6a)-(6c) where, for brevity, we have used the notations: $q = p_1/p_2$, and $r = (1 - p_1)/(1 - p_2)$.

(6b) $k = 0$, if $r^n < a_2/a_1$,

(6b) $k = n$, if $a_2/a_1 > r^{n-1}$, and

(6c) $r^{n-k}k > a_2/a_1 > r^{n-k+1}k-1$, with at least one strict inequality, for an interior optimum.

It is apparent that (6a) states the condition for a corner optimum under which no scrutiny of projects is desirable. Expression (6b), on the other hand, provides the condition under which the opposite extreme, entailing a
full unanimity, is desirable. In between these two extremes, the interior optimum is characterized by the expression (6c). A manipulation of (6c) yields the following results [see Appendix I(b) for derivations].

PROPOSITION 1:

\[(7a) \quad k < \frac{m}{2} + 1, \text{ if } a_1 > a_2, \text{ and } p_2 \leq 1 - p_1\]

\[(7b) \quad k > \frac{m}{2}, \text{ if } a_1 < a_2, \text{ and } p_2 \geq 1 - p_1\]

\[(7c) \quad \text{majority rule, if } a_1 = a_2, \text{ and } p_2 = 1 - p_1.\]

Expressions (7a) and (7b), respectively, delineate sufficient conditions under which the optimal consensus is smaller than, and larger than, that in the majority rule. Expression (7c) is simply the special case of (7a) and (7b), under which the majority rule is optimal.

To understand these results, recall that a larger \( k \) lowers the proportion of projects (good or bad) accepted by the committee. This is desirable if the project portfolio is relatively bad, but it is undesirable if the portfolio is relatively good. The quality of the project portfolio is captured in our model by the magnitude of \( a_1 \) relative to \( a_2 \). This relative magnitude represents how large is the proportion of good projects, and how large is the gain from a good project compared to the loss from a bad project.

The implications of the portfolio quality are seen clearly at the extremes. If the portfolio is extremely good, that is, if \( a_2 \) is negligible compared to \( a_1 \), then any scrutiny is entirely undesirable, and the committee's profit is maximized by setting \( k = 0 \). At the other extreme, if the project portfolio is extremely bad, that is, \( a_1 \) is negligible compared to \( a_2 \), then the maximum possible scrutiny is
desirable, and the profit is maximized by setting $k = n$. These two conclusions, in fact, can be observed directly from (6a) and (6b).

The aspect of individual's decision making capability which is central to our analysis is whether they are more likely (or less likely) to reject a good project than to accept a bad project. That is, whether their Type-I error, $1 - p_1$, is larger (or smaller) than their Type-II error, $p_2$. It is not surprising, thus, that the results (7a)-(7c) are dependent critically on the relative magnitude of these two types of errors.

Once again, extreme cases are illustrative. For instance, if individuals' Type-II errors are relatively negligible, that is, $1 - p_1 > p_2 \to 0$, then it is never desirable to have $k$ larger than one. This is because, if individuals reject all bad projects, then more than one screening of projects can only reduce the number of good projects accepted, which would reduce committee's profit. In contrast, if the Type-I errors are negligible, that is, $p_2 > 1 - p_1 \to 0$, then the profit is maximized by setting $k = n$. In this case, individuals do not reject any good project; additional scrutiny, therefore, can only improve the committee's profit because fewer bad projects would then be accepted.

When the effects of the portfolio quality and that of the individual's errors are combined together, then (7c) can be viewed as a benchmark condition for majority rule to be optimal. This happens when the quality of project portfolio is intermediate ($\alpha_1 = \alpha_2$), and the two types of individuals' errors are equal. With this benchmark result (7a) and (7b) are easily understood. If the project portfolio is better (worse), and an individual is more (less) likely to reject a good project than to accept a bad project, then the scrutiny should be slacker (tighter) than the majority rule.
B. Comparative Statics

In the above model, the optimal consensus depends on the parameters \((n, a_1, a_2, p_1, p_2)\). For a comparative statics analysis with respect to these parameters, we focus on an interior optimum, treat \(k\) and \(n\) as continuous variables, and employ the standard normal approximation to the binomial distribution entailed in (1). That is

\[
h_i = 1 - \Phi(z_i)
\]

where \(\Phi\) is the unit normal distribution function, and

\[
z_i = (k - np_i)/(np_i(1 - p_i))^{1/2}.
\]

The derivatives of (8) with respect to \(k\) and \(n\) are

\[
h_{ik} = -\Phi'(z_i)[np_i(1 - p_i)]^{-1/2} < 0
\]

and

\[
h_{kn} = \Phi'(z_i)\frac{1}{2n}(k + np_i)[np_i(1 - p_i)]^{-1/2} > 0.
\]

Expressions (9) and (10) are the continuous, and approximated, versions of (2) and (3) respectively. Their respective signs are identical, which is what we would expect. From (5), the optimum is characterized by

\[
y_k = a_1h_{1k} - a_2h_{2k} = 0, \text{ and } n > k > 0.
\]

If \(\theta\) represents an exogenous parameter, then a perturbation in (11) yields

\[
\frac{dk}{d\theta} = -\frac{y_{kk}}{Y_{kk}}.
\]

\(Y_{kk}\) evaluated at the optimum, using (9) and (11), is

\[
y_{kk} = b a_1 h_{1k}(p_1 - p_2)/np_1p_2(1 - p_1)(1 - p_2)
\]
where \( b = k(1 - p_1 - p_2) + np_1p_2 \). \( b \) can be reexpressed as
\[
k(1 - p_1)(1 - p_2) + (n - k)p_1p_2,
\]
which is positive. Thus, from (9) and (13), it is obvious that \( Y_{kk} < 0 \).

**Effect of Committee Size:** We first evaluate \( Y_{kn} \) at the optimum, and substitute the resulting expression, along with (13), into (12). This yields
\[
\frac{dk}{dn} = \frac{k^2(1 - p_1 - p_2) + n^2p_1p_2}{2nb}
\]

Now, note that the numerator in the above right hand side equals
\[
k^2(1 - p_1)(1 - p_2) + (n^2 - k^2)p_1p_2 > 0
\]
and, therefore, \( \frac{dk}{dn} > 0 \).

That is: A larger committee has a larger optimal consensus. This makes intuitive sense since, if the size of consensus is left unchanged but the committee size is increased, then clearly the scrutiny becomes slacker than before. More projects—including more bad projects—get approved. To restore the desired tightness in screening, then, it is reasonable that the required consensus should be increased.

Another bound on \( \frac{dk}{dn} \) is obtained by noting from (14) that:
\[
1 - \frac{dk}{dn} = \frac{k(2n - k)(1 - p_1)(1 - p_2) + (n - k)^2p_1p_2}{2nb} > 0.
\]

Thus: The optimal consensus increases less than the increase in the committee size. The reason for this is parallel to the argument used earlier. Assume that \( k \) is increased by the same number that \( n \) did. A project is now rejected if the same number of individuals disapprove. But since there are more individuals in the committee, it is easier for this to happen: the scrutiny has become unambiguously tighter. The appropriate response thus is to have a lower degree of scrutiny. Therefore \( k \) increases by less than the increase in \( n \).

To understand the role of individuals' errors in the present context,
we use (14) to obtain: \( \frac{dk}{dn} - \frac{1}{2} = k(k - n)(1 - p_1 - p_2)/2nb \geq 0 \), if \( p_2 \geq 1 - p_1 \). Clearly, \( \frac{dk}{dn} = \frac{1}{2} \), in the special case when \( p_2 = 1 - p_1 \).

This case could be called the marginal majority rule since the increase in the optimal consensus is one-half of the increase in the committee size.

Therefore: (i) The marginal majority rule is optimal if the two types of individuals' errors are equal, and (ii) The increase in the optimal consensus in response to an increased committee size is greater (smaller) than that in the marginal majority rule if individuals are less (more) likely to reject a good project than to accept a bad project.

These results are parallel to those in (7a)-(7c), and can be understood in a similar manner, keeping in mind one critical difference.

These results (7a)-(7c) are global, but they hold only when the quality of the portfolio satisfies certain conditions. In contrast, the present results hold only in the neighborhood of an interior optimum, but they do not depend on the quality of the project portfolio.

Next, we note a simple result on the effect of committee size on the optimal consensus as a proportion of the committee size. Consider the case where the two types of errors are equal, that is, \( p_2 = 1 - p_1 \).

Then, from above, \( \frac{dk}{dn} = \frac{1}{2} \). Also, when \((k, n)\) are treated as continuous variables, then (7a) and (7b) are equivalent to

\[
\frac{k}{n} \leq \frac{1}{2}, \text{ if } a_1 \geq a_2, \text{ and } p_2 = 1 - p_1.
\]

Thus, \( \frac{d(k)}{dn(n)} = \left( \frac{dk}{dn} - \frac{k}{n/n} \right) \geq 0 \), if \( a_1 \geq a_2 \).

Combining this result with (7a) and (7b) it follows that: When the two types of individuals' errors are equal, the optimal decision rule becomes closer to the majority rule as the size of committee increases. The above results are summarized in the following proposition.
PROPOSITION 2:

\[(16a) \quad 1 > \frac{d_k}{dn} > 0 \]

\[(16b) \quad \frac{d_k}{dn} \leq \frac{1}{2}, \text{ if } p_2 \leq 1 - p_1; \]

\[\text{marginal majority rule, if } p_2 = 1 - p_1 \]

\[(16c) \quad \left(\frac{d_k}{dn}\right)_n > 0, \text{ if } \alpha_1 < \alpha_2, \text{ and } p_2 = 1 - p_1. \]

*Effect of Portfolio Quality:* A larger \( \alpha \), a larger \( x_1 \), or a smaller \( x_2 \) implies that the portfolio faced by the committee is better. Further, from (11), \( \frac{\partial Y_k}{\partial \alpha} < 0 \), \( \frac{\partial Y_k}{\partial x_1} < 0 \), and \( \frac{\partial Y_k}{\partial x_2} > 0 \). Using (12) and recalling that \( Y_{kk} < 0 \), therefore, the effect of portfolio quality is immediately ascertained.

PROPOSITION 3: *The optimal size of consensus is smaller if a committee faces a better portfolio.*

This result is clearly in agreement with what we would have expected based on our earlier discussion.

*Effect of Managerial Quality:* An improvement in the individuals' decision making abilities is represented in our model by a larger \( p_1 \) and a smaller \( p_2 \). This, from (4), implies that a committee accepts more good projects, and rejects more bad projects. The impact of an improvement in the managerial quality on the optimal consensus, however, is ambiguous in general; under some circumstances, it may be desirable to reduce \( k \), so that a yet larger proportion of good projects can be accepted; whereas in other circumstances, it may be better to increase \( k \), so that the acceptance of bad projects is lowered even further.
To see this, consider the special case in which the two types of individual errors are equal; that is, \( p_2 = 1 - p_1 \). A higher \( p_1 \) now represents not only a lower Type-I error but also a lower Type-II error. In this case, it can be ascertained from (11) that \( \frac{\partial Y_k}{\partial p_1} \leq 0 \), if \( k \leq \frac{1}{2} \). Combining the last expression with (12) and (15), we obtain

\[
(17) \quad \frac{\partial k}{\partial p_1} \leq 0, \text{ if } a_1 \geq a_2, \text{ and } p_2 = 1 - p_1.
\]

This result has an interesting implication. If the project portfolio is relatively bad (that is \( a_1 < a_2 \)), then we know that a larger consensus is desirable. Now, if the managerial quality improves then, according to (17), the scrutiny should be slackened so that more good projects can be accepted. On the other hand, if the portfolio is relatively good (that is, \( a_1 > a_2 \)), and if the managerial quality improves, then the scrutiny should be tightened, so that a larger number of bad projects can be rejected. Thus: **When the two types of individuals' errors are equal, the optimal decision rule becomes closer to the majority rule as the managerial quality improves.**

III. THE OPTIMAL SIZE OF A COMMITTEE

In this section, we briefly consider the simultaneous determination of the optimal committee size and the optimal size of consensus required to accept projects; in particular, we look at the impact of the evaluation cost on the optimal \( k \) and \( n \). For this, once again, we adopt the approximation (8), and focus on an interior \((k, n)\). This is characterized by (11), and by

\[
(18) \quad Y_n = a_1 h_1n - a_2 h_2n - C_n = 0,
\]
which is the first order condition of optimality with respect to $n$. Let $\theta$ denote a parameter of the evaluation cost, such that a larger $\theta$ implies a larger marginal cost of committee members; that is $C_{n\theta} > 0$. Then, it is obvious from our earlier analysis that the optimal $k$ is affected by $\theta$ only through the change in $n$; that is, $\frac{dk}{d\theta} = \frac{dk}{dn} \frac{dn}{d\theta}$. Also, using the envelope theorem, and assuming that $Y_{nn} < 0$ at the optimum, a perturbation in (18) yields $\frac{dn}{d\theta} < 0$. Therefore, the results obtained in Proposition 2 can be translated immediately to ascertain the effect of evaluation cost on the optimal consensus, when the committee size is optimal. For instance, multiplying (16a) by $\frac{dn}{d\theta}$, we obtain the following proposition.

**PROPOSITION 4:** A larger marginal cost of committee members leads to a smaller committee size as well as to a smaller size of consensus. But, the former decline is larger than the latter decline.

IV. HIERARCHIES, POLYARCHIES, AND COMPLEX ORGANIZATIONS

A. Hierarchies

In many organizations, decisions are made sequentially. Consider a bureaucracy consisting of $n$ bureaus, in which a higher bureau (individual) examines only those projects which have been approved by the bureau below it, and only those projects are finally accepted by the organization which are approved by the highest bureau. This bureaucracy has some similarity to a committee in which unanimity is required; specifically, the probability of projects' acceptance by a hierarchy can be obtained by substituting $k = n$ into (1), which yields $h(p_i) = p_i^n$.

A key difference between the two organizational forms, however, is
that in the present case the number of evaluations that a project goes through depends not only on \( n \) (as it does in a committee) but also on bureaus' probabilities of acceptance of different projects. This is because larger acceptance probabilities imply that all bureaus (except the one which is lowest) must evaluate a larger number of projects. To see this, note that if \( p \) is the probability that a project is accepted by a bureau, then the expected number of evaluations for a project is:

\[
1 + p + \ldots + p^{n-1} = \frac{(1 - p^n)}{(1 - p)}.
\]

If one evaluation costs \( c \), then the expected evaluation cost per project is

\[
C(n, p_1, p_2) = cn\frac{(1 - p_1^n)}{(1 - p_1)} + c(1 - \alpha)\frac{(1 - p_2^n)}{(1 - p_2)}.
\]

(19)

It is then easily verified that \( \frac{\partial C}{\partial p_1} > 0 \). Also, as one would expect, the evaluation cost is larger in a larger hierarchy; that is, \( \frac{\partial C}{\partial n} > 0 \).

Substitution of (19), and of \( h(p_1) = p_1^n \), into (5) shows that the maximization of expected profit is equivalent to maximizing

\[
y^H = p_1^n - \beta p_2^n,
\]

(20) where

\[
\beta = (1 - \alpha)\left(\frac{z_2 - \frac{c}{1 - p_2}}{a}\right)/\left(\frac{z_1 + \frac{c}{1 - p_1}}{1 - p_1}\right).
\]

(21)

In (20), \( \beta \) can be viewed as a summary parameter representing the 'effective' portfolio quality; it is the relative loss in accepting a bad project (when the gain from accepting a good project is 1), taking into account the cost of evaluating good and bad projects. A smaller \( \beta \), thus, denotes a higher effective quality of the portfolio.

A meaningful trade-off between the Type-I and Type-II errors requires
that \( \beta > 0 \); otherwise all projects would be accepted by setting \( n = 0 \). From (21), \( \beta \) is positive if \( x_2 > x_2p_2 + c \); that is, if the expected loss (including the evaluation cost) from evaluating a bad project for the first time is smaller than the loss if the same bad project is accepted without evaluation. The evaluation of projects is clearly unnecessary if this condition is not met.

Treating \( n \) as a continuous variable, the first order condition of optimality of (20), with respect to \( n \), yields the following expression for the optimal number of levels in a hierarchy.\(^{13}\)

\[
H = \frac{\ln(\beta \ln p_2 / \ln p_1)}{\ln(p_1/p_2)}, \quad \text{and} \quad \frac{dn}{d\beta} > 0.
\]

Our interpretation of the parameter \( \beta \) suggests that the effective quality of the portfolio should be higher if the actual quality of the portfolio is higher. This is verified directly from (21), which yields:

\[
\frac{\partial \beta}{\partial a} < 0, \quad \frac{\partial \beta}{\partial x_1} < 0, \quad \text{and} \quad \frac{\partial \beta}{\partial x_2} > 0.
\]

Also, \( \frac{\partial \beta}{\partial c} < 0 \). Using (22), therefore, we obtain the following, easily understandable, result.

**PROPOSITION 5:** A better project portfolio, or a higher evaluation cost, implies a smaller number of levels in a hierarchy.

The quality of managerial decision making (represented by \( p_i \)'s) has a direct effect on the selection of projects, and also an indirect effect on the evaluation costs. The corresponding implications on \( n^H \) can be separated, respectively, as

\[
\frac{dn}{dp_i} = \frac{dn}{dp_i} + \frac{dn}{\partial \beta} \frac{\partial \beta}{dp_i}.
\]
The evaluation of the indirect effect is straightforward. As remarked earlier, a larger $p_1$ means a larger evaluation cost which, in turn, lowers the optimal number of levels in a hierarchy.\footnote{14}

The direct effect (through the selection of projects) is, however, ambiguous in general; in some cases, improved managerial quality (that is, a larger $p_1$ or a smaller $p_2$) may lower $n^H$, while in other cases it may raise $n^H$. This ambiguity is parallel to the one which was observed concerning the effect of managerial quality on the decision rules for a committee. Sufficient conditions can be obtained, however, under which the effect of $p_1$ on $n^H$ is predictable. For instance, it can be ascertained [see Appendix (c)] that: $\frac{\partial n^H}{\partial p_1} > 0$ if $\beta < 1$; and $\frac{\partial n^H}{\partial p_2} > 0$ if $\beta > 1$. That is: The direct effect of a higher Type-I (Type-II) managerial error is to lower (raise) the optimal number of levels in a hierarchy, if the effective quality of the portfolio is high (low).

B. Polyarchies

The hierarchical decision structure examined above requires complete unanimity. At the opposite extreme are polyarchical decision systems in which no consensus is required. Such decision making mechanisms are stylistically parallel to those decentralized institutions in which individual units make their decisions independently; a project is undertaken if any one of the units accepts it. The project acceptance, in this case, is similar to that in a committee in which adoption of a project requires only one member's approval. If $n$ is the number of units in a polyarchy, then substitution of $k = 1$ into (1) yields $h(p_1) = 1 - (1 - p_1)^n$.

The particular flow of projects on which we focus here is the one in
which a project arrives randomly at one of the units which evaluates the project. The project is evaluated by another unit only if the first unit rejects the project; this chain of evaluation continues until the project is accepted by any one of the units, or until it is rejected by all units. The same project, however, is not evaluated more than once by any one unit. The expected number of evaluations for a project then is:

\[ 1 + (1 - p) + \ldots + (1 - p)^{n-1} = [1 - (1 - p)^n]/p , \]  

and

\[ C(n, p_1, p_2) = c\alpha[1 - (1 - p_1)^n]/p_1 + c(1 - \alpha)[1 - (1 - p_2)^n]/p_2 . \]  

In contrast to (19), the evaluation cost in this case is lower if \( p_1 \) and \( p_2 \) are higher. This is intuitive because if one unit accepts more projects, then other units evaluate fewer projects.

Substituting the above cost function, and \( h(p_1) = 1 - (1 - p_1)^n \), into (5), it follows that the expected profit maximization is equivalent to maximizing

\[ x^p = -(1 - p_1)^n + \beta(1 - p_2)^n , \]  

where

\[ \beta = (1 - \alpha)(z_2 + \frac{\alpha}{p_2})/\alpha(z_1 - \frac{\alpha}{p_1}). \]

Once again, \( \beta \) summarizes the effective portfolio quality, taking into account the evaluation cost; but now \( \beta \) is the relative gain in rejecting a bad project, when the loss in rejecting a good project is 1. A smaller \( \beta \) implies a higher effective quality of the portfolio.

Parallel to (22), we obtain the optimal number of units in a polyarchy as

\[ n^p = \frac{\ln[\beta \ln(1 - p_2)/\ln(1 - p_1)]}{\ln(1 - p_1)/\ln(1 - p_2)} , \]  

and \( \frac{dn^p}{d\beta} < 0 \).
The analysis of the above expression is quite similar to that of (22); we therefore leave out the details, and summarize the results.

PROPOSITION 6: A better portfolio, or a smaller evaluation cost, implies a larger number of units in a polyarchy.

Also, given the symmetry between (22) and (26), it is obvious that the (direct and indirect) effects of \( p_i \) on \( n^P \) are precisely opposite to those on \( n^H \). Specifically, a larger \( p_i \) reduces the evaluation cost and, thus, raises the optimal number of units in a polyarchy. Also,

\[
\frac{\partial n^P}{\partial p_1} < 0 \text{ if } \beta < 1, \quad \text{and} \quad \frac{\partial n^P}{\partial p_2} < 0 \text{ if } \beta > 1.
\]

These sufficient conditions can be interpreted in a manner similar to that outlined earlier.

An analogy to the problem of resource extraction: An interesting, and useful, interpretation of the above analysis is as follows. If the set of good projects in the portfolio is viewed as the valuable resource then, for any organization, there are three costs of extracting this resource:

(i) evaluation costs, (ii) the loss due to inadvertent acceptance of bad projects, and (iii) the cost due to lost opportunities if some part of the portfolio (containing some good projects) is no longer available for evaluation. With this interpretation, it is clear that hierarchies increase the proportion of good to bad projects at successively higher levels but, in the same process, they deplete the stock of projects available for consideration. Polyarchies, in contrast, are better at preserving the portfolio of projects but they entail a higher loss due to the acceptance of bad projects.
C. Complex organizations

An implication of the above analogy is that a combination of hierarchical and polyarchical features might perform better under certain circumstances. As an illustration, consider a hierarchy consisting of \( n \) levels, and compare its performance to a polyarchy consisting of two hierarchies, each of which has \( \frac{n}{2} \) levels. Denote the variables corresponding to these two organizations by superscripts 1 and 2 respectively. Clearly, \( h^1 = p^n \), and \( h^2 = 1 - (1 - p^{n/2})^2 \). If the evaluation costs depend only on the number of managers, then

\[
Y^2 - Y^1 = 2\alpha_1(t_1 - t_2)(1 - t_1 - t_2) + 2(\alpha_1 - \alpha_2)t_2(1 - t_2)
\]

where \( t_1 = p^{n/2} \). Now, if \( n \) is large, then \( t_1 \) and \( t_2 \) are small, and \( 1 > t_1 + t_2 \). Thus, if \( \alpha_1 > \alpha_2 \), then \( Y^2 > Y^1 \). The same is true even if \( \alpha_1 < \alpha_2 \), provided \( \alpha_1 \) is not too small compared to \( \alpha_2 \).

Thus: If the evaluation costs depend only on the number of managers, and if the portfolio quality is not too low, then it is profitable to reorganize a large hierarchy into two hierarchical subunits within a polyarchy.

The above example of a mixed organizational form suggests two important questions: first, is it possible to design complex organizations (that is, a combination of committees, hierarchies and polyarchies; polyarchies; where each subunit is itself a committee, hierarchy or polyarchy; and so on) which attain perfect screening; and, second, are such organizations economically desirable? The answer to the first question is not only yes but, in fact, there are several alternative arrangements which, with sufficiently large number of screens, yield
perfect screening. The following two propositions describe two alternative ways [see Appendix I(d) for details].

PROPOSITION 7: An \((m, n)\) polyarchy-hierarchy (that is, an \(m\)-unit polyarchy where each unit is an \(n\)-level hierarchy) in which each level of hierarchy is itself an \((m, n)\) polyarchy-hierarchy will, with sufficient iterations, yield perfect screening.

PROPOSITION 8: A \((k, n)\) committee (that is, a committee consisting of \(n\) subcommittees, which adopts a project if it is approved by at least \(k\) subcommittees) in which each subcommittee itself is a \((k, n)\) committee will, with sufficient iterations, yield perfect screening.

The intuition underlying the above results is simple. A polyarchy consisting of units which are themselves hierarchies can be designed, by choosing appropriately the number of units in the polyarchy and the number of levels in the hierarchy, such that it has better screening ability (that is, it makes lower Type-I as well as Type-II errors) than an individual. This is because each of the hierarchies improve the ratio of good to bad projects, while the polyarchical structure allows a better preservation of the portfolio. Extending this logic, then, the screening ability can be further improved by treating the above polyarchy-hierarchy as a single subunit, and by constructing another hierarchy-polyarchy consisting of such subunits. When this process is repeated then, in the limit, perfect decision making ability is obtained. This is what Proposition 7 says; parallel intuition underlies Proposition 8.

Note that these propositions are independent of any assumptions concerning the nature of the errors in judgment which are made; all that
is required is that each individual have some discriminating ability, no matter how small; that is, \( p_1 > p_2 \). Another important point to be noted here is that increasing the number of individuals in an organization, in itself, does not yield perfect screening. For instance, it is trivial to show that adding more levels to a hierarchy, beyond some point, actually lowers the organization's performance; this is true even if the evaluation costs were zero.

Of course, we do not see perfect screening; nor would we expect to see it in economic organizations. The reason is obvious: there are costs associated with evaluation, and perfect screening requires such large amounts of resources to be spent on evaluation that it is economically undesirable. That is precisely why we have emphasized evaluation costs in our analysis. The presence of evaluation costs means not only that all organizations are fallible in their decisions (like the individuals of which they are composed) but also that (even taking, as we have done here, the individuals' errors as exogenous) the level and the nature of organizational errors are endogenous consequences of how the decision making is organized.

V. CONCLUDING REMARKS

This paper has analyzed the optimal structure of committees, hierarchies, and polyarchies, under some stylized assumptions. Some of these assumptions may easily be dropped; others would require substantial modifications to the analysis. First, we have ignored the costs and benefits of intra-organizational communication. One of the well observed facts of large committees, for instance, is that each
individual attempts to express his view. Individuals evidently believe that they have information which is not adequately summarized by their yes–no vote, and that this information may alter the beliefs (and hence the votes) of other individuals in the committees. Such communication has several effects. It requires time to communicate. If it takes $\tau$ units of time for one individual to communicate his information, and if an individual communicates simultaneously to the entire committee, then $m \tau$ units of time are used by an $n$ member committee. If, on the other hand, communications are bilateral, then the time taken is $m(n-1)\tau$, which rises much more rapidly with the committee size.

Also, the benefits of communication may diminish as the committee size increases; it is more likely that an individual's information is spanned by the information that others have. The increase in the information to be obtained from communication, therefore, would be less than proportionate as the committee size increases. A related aspect is the error in communication: individuals can never communicate fully their information, and the information received may be quite different from the one which was intended by the person communicating it.

Second, we have abstracted from important issues related to incentives, many of which have been treated extensively elsewhere. A key aspect to which we should call attention, however, is that a manager's decision to pass on a project to a higher level for a review imposes costs on the organization; most organizations do not charge the lower level managers directly for these costs; designing incentive structures and organizational rules which serve to internalize these externalities is a question we hope to pursue elsewhere. 

Third, we have assumed that individuals are homogeneous in their
decision making abilities. Since committees involve symmetric decision rules, they make more sense when in fact individuals are homogeneous. Hierarchies, polyarchies and complex organizations, on the other hand, may have advantages over committees (in ameliorating individuals' errors) when different individuals have different abilities; that is, when they make different types of errors with different frequencies. With heterogeneous individuals, the problems of self-perpetuating organizations — like boards of directors of foundations and tenured faculties at universities — who choose their successors become important. Clearly, then, different types of organizations give rise to different probability distributions of abilities among the next generation's members and, thus, to different stochastic processes of ability distribution over time. In our 1985b paper, we have analyzed this problem of self-perpetuating organizations.
(a) Substituting (1) into (5), it can be verified that
\[ Y(k) - Y(k - 1) = a_1 [Y(k + 1) - Y(k)] + a_2, \]
where \( a_1 \) and \( a_2 \) are positive numbers. The last expression can be rearranged to yield

(28) \[ \text{If } Y(k) \geq Y(k - 1), \text{ then } Y(k - 1) > Y(k - 2), \text{ and} \]

(29) \[ \text{If } Y(k) \geq Y(k + 1), \text{ then } Y(k + 1) > Y(k + 2). \]

The above expressions imply that \( Y \) is single peaked in \( k \). This can be seen as follows. If \( Y(k = 0) \geq Y(k = 1) \), then, from (29),
\[ Y(k = 0) > Y(k > 1), \text{ and } k = 0 \text{ is optimal. If} \]
\[ Y(k = n) \geq Y(k = n - 1), \text{ then, from (28), } Y(k = n) > Y(k < n - 1), \]
and \( k = n \) is optimal. If \( Y(k = 1) > Y(k = 0) \), and
\[ Y(k = n - 1) > Y(k = n), \text{ then there is an interior optimum at } k, \text{ such} \]
that \( Y(k) \geq Y(k - 1) \) and \( Y(k) \geq Y(k + 1) \), with at least one strict inequality. Using (1) and (5), the above three results can be restated as (6a), (6b), and (6c), respectively.

(b) Since, \( q = p_1/p_2 \), and \( r = (1 - p_1)/(1 - p_2) \), it follows that

(30) \[ rq \geq 1, \text{ if } p_2 \leq 1 - p_1. \]

Next, rewrite (6c) as

(31) \[ (n - k)lnrq + (2k - n)lnq \geq \ln(a_2/a_1) \]
\[ \geq (n - k + 1)lnrq + (2k - n - 2)lnq. \]

Now, suppose (7a) is not true; that is, \( k \geq \frac{m}{2} + 1 \), when \( a_1 > a_2 \) and \( p_2 \leq 1 - p_1 \). Then, the right hand side of (31) is nonnegative, since
rq ≥ 1 from (30), and (2k - n - 2) ≥ 0. On the other hand, 
ln(a_2/a_1) < 0. Expression (31) is thus contradicted. Using a parallel argument, it can be established that (31) is contradicted if (7b) is not true. Expression (7c) is simply a special case of (7a) and (7b). It provides a sufficient condition for the majority rule to be optimal

$$\frac{n}{2} + 1 > k > \frac{n}{2}, \text{ if } a_1 = a_2, \text{ and } p_2 = 1 - p_1.$$ (32)

If n is an odd number, then the majority rule implies: k = (n + 1)/2.

If n is an even number, then the two candidates for the majority rule are k = n/2, and k = n/2 + 1. It is easily verified that both of these solutions yield the same profit.

(c) Expression (22) yields

$$\frac{\partial n}{\partial p_1} = g_1[-1 + \ln \beta w + w], \text{ and}$$ (33)

$$\frac{\partial n}{\partial p_2} = g_2[-1 + \ln \beta w + \frac{1}{w}]$$ (34)

where g_1 and g_2 are positive numbers, and w = ln p_2/ln p_1 > 0. Next, note that ln(·) is strictly concave in its argument. Thus,

$$\beta w - 1 > \ln \beta w > 1 - 1/\beta w.$$ (35)

Substitution of the left part of the above inequality into (33) yields

$$\frac{\partial n}{\partial p_1} > 0 \text{ if } 1 > \beta. \text{ Similarly, substitution of the right part of the}$$

inequality (35) into (34) yields $$\frac{\partial n}{\partial p_2} > 0 \text{ if } \beta > 1.$$ (d) Let h^1(m, n, p) denote the probability of a project's acceptance in an (m, n) polyarchy-hierarchy. Then h^1(m, n, p) = 1 - (1 - p^n)^m,
\( h^1_p > 0 \), \( h^1_m > 0 \), and \( h^1_n < 0 \). Now, choose \( m \) and \( n \) such that 
\[ h^1(m, n, p_0) = p_0, \] 
where \( p_1 > p_0 > p_2 \). Thus, \( h^1(p_1) > p_1 \), and
\[ h^1(p_2) < p_2; \] and hence the \((m, n)\) polyarchy-hierarchy has lower
Type-I as well as lower Type-II errors than a single individual. Next,
treat the above \((m, n)\) polyarchy-hierarchy as a single subunit, and
construct an \((m, n)\) polyarchy-hierarchy consisting of such subunits. If
the variables corresponding to the latter organization are denoted by the
superscript 2; that is, \( h^2 = 1 - (1 - (h^1)^n)^m \); then
\[ h^2(p_1) > h^1(p_1) > p_1, \] and \( h^2(p_2) < h^1(p_2) < p_2 \). Thus, the two
types of errors are reduced even further. If this process is iterated
then, in the limit, perfect screening of projects is achieved. Similar
reasoning underlies Proposition 8. In fact, its proof is identical to
that of Moore-Shannon theorem in reliability literature [see Harrison
(1965, pp. 255–262), for example].
FOOTNOTES

1. There is a resemblance between this characterization and some of the problems studied in the reliability theory; a fallible manager evaluating a portfolio containing two types of projects can be viewed as a relay network's component subject to two kinds of failures. However, the results obtained here are, to our knowledge, not available in the reliability literature.

2. This simple representation of the project portfolio allows us to focus sharply on the questions outlined earlier. A more general portfolio, consisting of a continuum of projects, as well as the possibility that the portfolio may itself be affected by the committee's structure (for example, due to the effect of committee's decisions on the incentives of project inventors) can be modelled along the lines of Sah and Stiglitz (1984).

3. This can be derived as follows. The probability that a committee accepts a project, given that one of its members has accepted the project is \( h(k - 1, n - 1) \). The probability that a committee accepts a project, given that one of its members has rejected the project is \( h(k, n - 1) \). Clearly, then,

\[
h(k, n) = ph(k - 1, n - 1) + (1 - p)h(k, n - 1) .
\]

This expression, in combination with (2), can be rearranged to yield (3).

4. Specifically, \( \frac{\partial h}{\partial p} = \binom{n}{k}kp^{k-1}(1 - p)^{n-k} > 0 \), which can be obtained from (1).
5. Note here that, since \( p_1 > p_2 \), conditions such that \( p_2 \) is greater (or smaller) than \( 1 - p_1 \) imply certain restrictions on the magnitudes of \( p_1 \) and \( p_2 \). Specifically, \( p_2 > 1 - p_1 \) means that \( p_1 > 1/2 \), whereas \( p_2 < 1 - p_1 \) means that \( p_2 < 1/2 \). Naturally then, \( p_2 = 1 - p_1 \) means that \( p_1 > 1/2 > p_2 \).

6. All subscripts, other than 1, 2 and \( i \), denote the variables with respect to which a partial derivative is being taken.

7. A simple derivation of (13) is as follows. Expression (11) can be reexpressed as \( Y_k = a_1 h_{2k}[(h_{1k}/h_{2k}) - (a_2/a_1)] = 0 \) which, upon differentiation, yields \( Y_{kk} = a_1 h_{2k} \frac{\partial}{\partial k}(h_{1k}/h_{2k}) \). Expression (9) is then used to obtain (13). The same method is helpful in deriving \( Y_{kn} \) and \( \frac{\partial Y_k}{\partial p_1} \), which are needed below.

8. \( Y_{kn} = -a_1 h_{1k}(p_1 - p_2)[k^2(1 - p_1 - p_2) + n^2 p_1 p_2]/2n^2 p_1 p_2(1 - p_1)(1 - p_2). \)

9. Another way to express this result is the following. Let \( a_{kn} = \frac{dnk}{dnm} \) denote the elasticity of the optimal consensus with respect to the committee size. Then: \( a_{kn} \geq 1 \) if \( a_1 \geq a_2 \), and \( p_2 = 1 - p_1 \). In addition, the lower bound on \( a_{kn} \) can be identified by noting from (14) that \( \frac{dk}{dn} - \frac{k}{2n} > 0 \). Thus, \( a_{kn} > 1/2 \), regardless of the relative magnitudes of the two types of individuals' errors. That is: The elasticity of optimal consensus with respect to the committee size is greater than one-half.

10. Since \( \frac{\partial Y_k}{\partial p_1} = -a_1 h_{1k}(n - 2k)[(1 - p_1)^2 + p_1^2]/2p_1^2(1 - p_1)^2. \)

11. Note, however, that when \( p_1 \) approaches unity, one would be indifferent among the decision rules (including the majority rule) since a perfect selection of projects is possible.
12. To ascertain this, denote the number of evaluations per project by
\[ v = (1 - p^n)/(1 - p). \]
Then \[ \frac{\partial v}{\partial p} = \left[ 1 + \left( n - \frac{n}{p} - 1 \right) p^n \right]/(1 - p)^2. \]
Let \( v_1 \) denote the numerator in the last expression. Then,
\[ \frac{\partial v_1}{\partial p} = -n(n - 1)(1 - p)p^{n-2} < 0, \text{ if } n > 1. \] Thus, \( v_1 \) attains its lowest value at \( p = 1 \), where \( v_1 = 0 \). Hence \( \partial v/\partial p > 0 \) if \( p < 1 \), and \( n > 1 \). In the case of a single bureau \((n=1)\), of course, \( v \) is independent of \( p \).

13. It is easily verified that the second order condition is satisfied at the optimum.

14. That is, \[ \frac{\partial^2 v}{\partial p_1^2} < 0, \text{ from (22).} \] Hence \[ \frac{\partial^2 h}{\partial \beta \partial p_1} < 0. \]

15. If \( n \) is sufficiently large then, in fact, it is desirable to break up a large hierarchy into more than two parts, under the conditions mentioned above. Also, it is easy to verify a parallel result that: Reorganization of a large polyarchy into two or more polyarchic subunits within a hierarchy yields a larger profit, if the portfolio quality is not too high and, once again, if the evaluation costs depend only on the number of managers.

16. These propositions can be easily extended to a project portfolio containing a continuum of projects.

17. The concept of 'first best' (that is, of perfect decision making) is, thus, not very useful in the present context.
18. Some aspect of communication have been examined by Klevorick, Rothschild and Winship (1984) in the context of jury decision making, where they compare majority rule without communication to the case where the observations of different individuals are aggregated using specific aggregation processes.

19. For additional remarks on the relationship between incentives and organization, see our 1984 paper. Also, we have abstracted in this paper from the costs and benefits of acquiring and processing information. There are, however, some simple statistical models for which the analysis presented here can be viewed as Bayesian. For an example of an explicit Bayesian framework, see our 1984 paper.
REFERENCES


