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DISTRIBUTION OF RENTS AND GROWTH

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Distribution of Rents and Growth

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ABSTRACT

This paper constructs a very simple model to investigate the effects of the intergenerational distribution of rents on saving, growth and fiscal policy. The rents in this model are generated from an investment externality. We consider a model in which the population grows at a constant rate, with new generations unrelated to previous ones. The model considered can have sustained growth in output per capita or can converge to a steady-state, depending on parameter values. The distribution of rents between labor and owners of firms affects the rate of growth in both cases.

In searching for explanations of the productivity slowdown in the U.S. in recent decades, Romer (1987) notes that there appears to be a large difference between the effects on output of adding a unit of capital (or labor), and capital's (or labor's) share in national income. The returns to factors do not reflect their marginal productivity, but instead may largely be determined by the share of external rents captured by the factors. The implications of various investment externalities for growth have, of course, been studied in great detail in recent years. However, we argue that the distribution of rents per se will have important effects in determining the growth path of the economy. In turn, the potential tradeoffs between growth and distributional goals are paramount in fiscal policy decisions. Indeed, the history of fiscal policy in the U.S. in the 1980s and 1990s appears to be one predicated on the conflict between groups trying to maintain their economic status in the face of real changes in the economy. Important changes in budgetary and tax policy have evolved in response to shocks to the economy which have led to a redistribution of returns to various groups. But there is a strong interdependence between the distributional goals of fiscal policy and the dynamic goals for saving, investment and growth.

The recent literature (Romer (1986, 1987), Grossman and Helpman (1991), for example) provides a useful starting point for investigating these issues. It has suggested that there are important external effects of investment. For example, if one firm increases expenditure on research and development, it is unlikely to be able to capture fully the social benefit of that investment. There is likely to be a positive spillover to other firms in the industry and in other sectors of the economy. Our focus is on the rents derived from the spillovers. In the absence of first-order debt neutrality, the distribution of the rents have an influence on the dynamic behavior of the economy. The level of aggregate consumption and the rates of capital accumulation and income growth depend on how the external rents are distributed.

In this paper, we will demonstrate in a simple example economy how the distribution affects these macro aggregates over time, and how the effects of fiscal policy depend on the distribution of the rents. Our goal in part is to develop a set of predictions from the model that might be tested, and would ultimately refine our understanding of the nature of economic growth.

In the existing literature, there has been little focus on the distributional effects of the growth externality. The bulk of the literature is set in a context of a single representative consumer. It is irrelevant in that type of model how the external rents are distributed. More recently, Saint-Paul (1992) and Alogoskoufis and van der Ploeg (1991) have examined overlapping-generations models with investment externalities, but ones in which all of the external rents accrue to labor. Thus, the distribution issue does not arise in their models by assumption.

Perotti (1990) and Persson and Tabellini (1991) have looked at the effect of the intragenerational size distribution of income on growth. Alesina and Rodrik (1991) and Bertola (1991) have focused on the intragenerational distribution of factor rewards. In contrast, we study the intergenerational distribution of external rents.

The extent to which each group captures these rents will decide the choice of fiscal policy for a government maximizing some social welfare function. In the presence of an investment externality such as the one described, a Pigouvian subsidy to investment can lead to a Pareto improvement in welfare. Even with such a subsidy in place, the government may wish to pursue goals that involve redistribution across generations.[1] Many of the issues that are raised in public debate concerning fiscal policy are not matters that involve policies that raise everybody's welfare. A policy that raises national saving usually redistributes from current (particularly older) generations towards future generations.

I. The Model

The model used in this paper is an overlapping generations of the Weil (1989) type. There is constant proportional population growth. Individuals have infinite horizons, and perfect foresight. All agents are endowed with identical human capital when they are born, and they have identical preferences. We aggregate the equations describing the behavior of optimizing individuals to obtain the dynamic behavior of the economy. (This contrasts with the older literature on optimal growth models with population growth, which studied the path for the economy when a social planner maximized per capita utility.) It is assumed that no individual's welfare depends directly on the utility of any other individual (i.e., no operative bequest motive).

This type of model shares many of the characteristics of other overlapping generations models. (In fact, algebraically, it is essentially identical to the Blanchard (1985) model.) In particular, the timing of lump-sum taxes matters. If taxes are deferred to later, unborn generations will

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[1] In the model we use, lump-sum redistributive fiscal policy cannot, in itself, improve everybody's welfare.

bear a larger share. Thus, government surpluses and deficits redistribute the tax burden between generations. Indeed, Buiter and Kletzer (1990) have shown that in overlapping-generations models any equilibrium generated by a particular sequence of budget deficits (or surpluses) and age dependent lump-sum taxes or transfers can be replicated by some other sequence of budget deficits (or surpluses) and only age independent lump-sum taxes or transfers.

In this section, we will begin by describing the consumers' optimization problems. Then, we will turn to the firm. Finally, we will set up the dynamic model in terms of aggregates.

Our notation convention in this paper is that variables pertaining to individual consumers or firms are in small letters, aggregate variables are in capital letters with a ~ over them, and aggregate per capita variables are in capital letters.

Consumers

Individuals are assumed to have logarithmic utility and maximize utility over an infinite horizon. The dynamic budget constraint faced by an individual at time t who was born at time v is

$$\dot{a}(v,t) = r\cdot a(v,t) + w(t) - \tau(t) - c(v,t).$$

The individual also faces the lifetime constraint

$$\lim_{t\to\infty} a(v,t)e^{-rt} = 0.$$

Finally, we assume that $a(v,v) = 0$.

In the above equations, $a(v,t)$ is the assets held by the individual at time t, and $c(v,t)$ is her consumption. $w(t)$ is the wage paid at time t. Note that it is independent of the individual's age. $\tau(t)$ are lump-sum taxes (or transfers if negative). We also assume that these are independent of age. Given the result of Buiter and Kletzer cited above, we do not lose any generality by considering only age-independent taxes and transfers, as long as we allow government budget deficits that satisfy the conventional solvency constraint. Furthermore, we assume $w(t) - \tau(t) \geq 0$. Finally, note that we treat the interest rate, r, as a constant. It will turn out that given our assumptions about firms' technology, the interest rate will always be constant.

It is easy to show that the optimal consumption choice is given by

$$c(v,t) = \delta[a(v,t) + h(t)],$$

where

$$h(t) = \int_t^{\infty}[w(s)-\tau(s)]e^{-r(s-t)}ds,$$

and δ is the rate of time preference.

A crucial assumption in this framework is that agents cannot trade claims to human capital. We will make a distinction between assets which can be traded (equity, bonds and capital) and those that cannot (human capital).

We assume $r > \delta$ throughout. Otherwise, in equilibrium consumers will just consume their labor income and never save. This is an uninteresting case.

Firms

Turning to the firm, we assume a very specific production function. We have that the output of firm j at time t is given by

$$y(j,t) = \theta k(j,t)^{\gamma}\ell(j,t)^{(1-\gamma)\eta}\tilde{K}(t)^{1-\gamma}\tilde{L}(t)^{-(1-\gamma)\eta} + \omega\ell(j,t).$$

In this equation, $y(j,t)$ refers to output by firm j at time t, $k(j,t)$ is that firm's capital input, and $\ell(j,t)$ is that firm's labor input. $\tilde{K}(t)$ is the aggregate capital stock, and $\tilde{L}(t)$ is the population at time t. (It is assumed that labor is inelastically supplied.) Note that the parameter ω is different than the wage rate, $w(t)$.

The production function is obviously a very special one. We chose this function because it yields an analytically tractable economic system, while capturing all of the essential elements of the problem we are interested in. The most important aspects of this function arise from our assumption that there are a constant number of firms in this economy, normalized to equal one. We are assuming, then, that there is some specific factor which is available to the firm and is not reproducible. Hence, the firm will have a positive value (more on this in just a moment). In equilibrium, since all firms are identical, we have $k(j,t) = \tilde{K}(t)$, and $\ell(j,t) = \tilde{L}(t)$. This implies that there is an aggregate production function given by $\theta\tilde{K}(t) + \omega\tilde{L}(t)$. The linearity of the aggregate function ensures that the real interest rate is constant over time.

This production function captures the notion that there is a positive externality to greater investment by the firm. The social marginal product of capital is θ, but the private marginal product is less than θ.

Labor's private marginal product is greater than its social marginal product, ω. The share of the externality that falls to labor (as opposed to owners of the firm) is measured by the parameter η. The larger is η, the greater the payment to labor.

A useful way to highlight some key features of our production function is to contrast it to the one considered by Saint-Paul (1992) and Alogoskoufis and van der Ploeg (1991). They have $\eta = 1$, and $\omega = 0$. Their first assumption implies that labor captures all of the external benefits from the investment externality. Since we argue that the interesting thing about fiscal policy in a model with investment externalities arises from the distributional effects of the externality, we clearly do not want η to equal one. We assume $\omega \neq 0$ because otherwise individuals' net worth at birth would be non-zero only because of the investment externality. If we were to consider the case in which all the rents accrue to owners of the firm, and insisted on setting ω=0, then the model would have the uncomfortable feature that individuals have no income, and thus can never save or consume.

The share of the externality that accrues to owners of the firm is given by $1-\eta$. Note that the firm would have no value were it not for the externality. One might wish to think of firms having property rights that allow them to exploit the externality. It would be simple enough to enter a linear term as we did for labor, so that there is a positive social return to a specific factor such as land. However, this would make the aggregate model very difficult to analyze, unless the function were linear in the quantity of the specific factor endowed to each firm divided by population. Such a specification seems hard to justify, and there does not seem to be any serious harm done in assuming the social marginal product of the specific factor is zero (as opposed to the analogous assumption for labor).

The firm chooses the capital stock and the labor input to maximize profits each period. There are no costs to installing capital, so the firm will set the marginal product of capital equal to the rental rate:

$$\theta\gamma k(j,t)^{\gamma-1}\ell(j,t)^{(1-\gamma)\eta}\tilde{K}(t)^{1-\gamma}\tilde{L}(t)^{-(1-\gamma)\eta} = r - \rho.$$

Here, ρ is the per unit of capital subsidy from the government, and is the only distortionary tax or subsidy in the model. In product market equilibrium, the above condition reduces to

$$\theta\gamma = r - \rho.$$

The condition that the marginal product of labor equal the wage rate is given in equilibrium by:

$$\theta\eta(1-\gamma)K(t) + \omega = w(t).$$

Note that this equation uses the aggregate *per capita* capital stock, K(t).

The stream of dividend payments for this firm are given by the value of output less the cost of factor payments (plus the subsidy to capital). Letting the price of the good be unity, we find in equilibrium that dividend payments are given by $\theta(1-\gamma)(1-\eta)\tilde{K}(t)$.

It is interesting to observe the factors that affect wages and dividends, given the capital stock. First, wages and dividends are higher the higher is the externality (the greater is $1-\gamma$). Second, wages are higher and dividends lower the greater is η. Note that dividends would be zero either if there were no externality ($\gamma=1$) or all the rents went to labor ($\eta=1$).

Financial Market Equilibrium

Before examining the aggregate system, we consider the financial market. Because all individuals have perfect foresight, all assets must pay the same rate of return. This implies that the return on government debt must equal the rental rate of capital, r.

It also means that the return to holding equities must equal r. The rate of return for equities is given by

$$r = \dot{\tilde{q}}(t)/\tilde{q}(t) + \theta(1-\eta)(1-\gamma)\tilde{K}(t)/\tilde{q}(t) - \tilde{P}(t)/\tilde{q}(t).$$

In this expression, $\tilde{q}$ represents the price per share. We assume there is one share per firm. Along with the assumption of the representative firm, $\tilde{q}$ can be seen to also equal the aggregate value of shares. Non-distortionary taxes on dividends are given by $\tilde{P}(t)$. So, we have that the capital gains rate plus the flow of dividends per share equals the interest rate.

We will assume dividends are always non-negative. That is,

$$\tilde{P}(t) \leq \theta(1-\eta)(1-\gamma)\tilde{K}(t).$$

Aggregate Values

We assume that population grows at the constant rate n. It is straightforward to aggregate as in Weil (1989). We will leave the details to the reader.

Aggregate accumulation per capita of tradeable assets is given by

$$\dot{A}(t) = (\theta\gamma+\rho-n)A(t) + w(t) - \tau(t) - C(t).$$

Recall that the interest rate, r, equals $\theta\gamma+\rho$.

We have that A(t) = K(t) + q(t) + D(t). K(t) is the aggregate per capita capital stock, $q(t)$ is the aggregate per capita value of shares, and $D(t)$ is the aggregate per capita government debt.

Consumption evolves according to

$$\dot{C}(t) = (\theta\gamma+\rho-\delta)C(t) - \delta nK(t) - \delta nq(t) - \delta nD(t).$$

The evolution of equity prices is given by:

$$\dot{q}(t) = (\theta\gamma+\rho-n)q(t) - \theta(1-\gamma)(1-\eta)K(t) + P(t).$$

The government budget constraint is given by:

$$\dot{D}(t) = (\rho+\gamma\theta-n)D(t) + \rho K(t) - T(t).$$

$T(t)$ is the sum of the two types of non-distortionary taxes: $T(t) = \tau(t) + P(t)$. The equation says that accumulation of debt equals interest payments on the debt (recalling $r = \rho + \gamma\theta$), plus subsidies to firm's for renting capital ($\rho K(t)$) less non-distortionary taxes. We also impose the government solvency constraint:

$$\lim_{t\to\infty} D(t)e^{-(r-n)t} = 0.$$

Taking the equations for $\dot{q}(t)$ and $\dot{D}(t)$ and subtracting them from the equation for $\dot{A}(t)$ (and using the fact that $w(t) = \theta\eta(1-\gamma)K(t) + \omega$) we have

$$\dot{K}(t) = (\theta-n)K(t) - C + \omega.$$

The four equations for $\dot{K}(t)$, $\dot{D}(t)$, $\dot{q}(t)$ and $\dot{C}(t)$ constitute a four equation dynamic system. However, the system is not complete until the fiscal policy choice is specified. A path for total non-distortionary taxes, $T(t)$, and for the amount of non-distortionary taxes that falls on dividends, $P(t)$, must be specified before the system is closed. In the next two sections we will examine the model under several different assumptions about fiscal policy. The variables $K(t)$, $D(t)$, $q(t)$ and $C(t)$ will constitute state variables under all the fiscal policy regimes we consider. For now, notice that the $\dot{K}(t)$ and $\dot{C}(t)$ equations are not affected by the values for T(t) and P(t).

II. The Laissez-Faire Economy

In this section, we will analyze the model when there are no taxes, no transfers, and no government debt -- a laissez-faire government. We are interested in characterizing how capital accumulation and consumption are affected by the distribution of rents and by the size of the externality.

When ρ, P(t), T(t), $\tau(t)$ and D(t) all equal zero at all times, the dynamic system can be written:

$$(1) \qquad \begin{bmatrix} \dot{C}(t) \\ \dot{K}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} \theta\gamma-\delta & -\delta n & -\delta n \\ -1 & \theta-n & 0 \\ 0 & -\theta(1-\gamma)(1-\eta) & \theta\gamma-n \end{bmatrix} \begin{bmatrix} C(t) \\ K(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \\ 0 \end{bmatrix}.$$

Recall that we are only examining the interesting case of $\theta\gamma-\delta > 0$.

A. The Effects of Rent Distribution

In this model, because labor is supplied inelastically and the number of firms is fixed, the share of rents accruing to each recipient does not affect any factor supply or factor demand decision. It only affects the supply of wealth across generations. The larger the share of rents that go to labor, the smaller the wealth of currently alive generations. When rents accrue to labor, future generations will capture some of those rents, but when rents accrue to firms they are captured by the current owners of the firm.

We will compare two economies in which the share of rents going to labor differ. Each economy has the same initial capital stock. In one economy, all rents are earned by labor[2]: $\eta = 1$. In the comparison economy, infinitesimally less of the rents accrue to labor, and go to firms instead. The choice of $\eta = 1$ as the benchmark economy is made because it simplifies the algebra, and is not critical for any of our conclusions.

There are three state variables which describe the path of the economy. $K(t)$ is predetermined, while $C(t)$ and $q(t)$ are not. Consumption and the value of firms can jump to place the economy on the saddle path. There need not be convergence to the steady state. The economy proceeds along the saddle path in both the stable and unstable cases. When all roots are positive, the economy is on the unstable saddle path. The fact that the unstable solution is feasible does not arise because of the overlapping generations nature of the model, or because of the externality. It is because the aggregate production function is linear. Even in the representative agent version of this model with no externality, the optimal path could be divergent, and capital and labor could grow indefinitely.[3] This is the case of sustained growth that has received considerable attention in the new growth literature.

We will assume that the interest rate is less than the long run growth rate. This assumption insures that the price of equities is always positive. In the stable case, the long-run growth rate of aggregate output is n, so the assumption implies $\theta\gamma > n$. We actually need a stronger restriction on the parameter space than this in the unstable case because the economy grows at

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[2] As in the papers of Alogoskoufis and van der Ploeg (1991) and Saint-Paul (1992).

[3] The model of Jones and Manuelli (1990) is one that emphasizes the role of non-concavity in production as a source for sustained growth.

the rate $\lambda+n$, where λ is the value of the smallest of the characteristic roots of the dynamic system. Hence, we will need that $\theta\gamma-n-\lambda > 0$.

Our strategy will be to examine the economy near the point where $\eta = 1$. This is convenient, because when $\eta = 1$, $q = 0$, and dynamic system is only 2x2 (two state variables, K and C, and two dynamic equations). So, we will end up evaluating derivatives like $\frac{dC(t)}{d\eta}$, and $\frac{dK(t)}{d\eta}$ from the 3x3 system evaluated at $\eta = 1$. The assumption that $\theta\gamma-n-\lambda > 0$ is accomplished in this case when $\delta n+\theta(\delta-n)(1-\gamma) > 0$.

The steady-state values of K(t), C(t) and q(t) are denoted $\bar{K}$, $\bar{C}$ and $\bar{q}$. Their values are:

$$\bar{K} = \frac{(\theta\gamma-\delta)(\theta\gamma-n)\omega}{\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)+\delta n\theta(1-\gamma)(1-\eta)}.$$

$$\bar{C} = \frac{[\delta n(\theta\gamma-n)+\delta n\theta(1-\gamma)(1-\eta)]\omega}{\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)+\delta n\theta(1-\gamma)(1-\eta)}.$$

$$\bar{q} = \frac{\theta(\theta\gamma-\delta)(1-\gamma)(1-\eta)\omega}{\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)+\delta n\theta(1-\gamma)(1-\eta)}.$$

The product of the characteristic roots (given by the determinant of the matrix in equation (1)) is equal to minus the denominator of each of the expressions for the steady-state values. Thus, when the system is saddle-path stable (one negative root and two positive), all of the variables are positive in steady state. When all roots are positive (the sustained growth case) then all the variables are negative in steady state.

The change in the steady-state values for a change in η are given by:

$$\frac{d\bar{K}}{d\eta} = \frac{\delta n\theta(1-\gamma)(\theta\gamma-\delta)(\theta\gamma-n)\omega}{[\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)+\delta n\theta(1-\gamma)(1-\eta)]^2} > 0.$$

$$\frac{d\bar{C}}{d\eta} = (\theta-n)\frac{d\bar{K}}{d\eta} > 0.$$

$$\frac{d\bar{q}}{d\eta} = \frac{-\theta(\theta\gamma-\delta)(1-\gamma)[\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)]\omega}{[\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)+\delta n\theta(1-\gamma)(1-\eta)]^2}.$$

When $\eta = 1$, the term in brackets in the numerator of $\frac{d\bar{q}}{d\eta}$ is equal to minus the product of the eigenvalues of the 2x2 system. Thus, in the stable case $\frac{d\bar{q}}{d\eta} < 0$, and in the unstable case $\frac{d\bar{q}}{d\eta} > 0$. We will only need to evaluate this derivative when $\eta = 1$.

There are an infinite number of dynamic paths that solve equation (1). Only one path (in the stable and unstable cases) satisfies the feasibility conditions for the economy that K(t) and C(t) remain positive in all time periods. Along this path, the variables converge to the steady state at a rate given by λ, the negative eigenvalue, if the system is stable. If it is unstable they diverge from the steady state at the rate λ, the smaller of the positive roots in the 2x2 system.[4] We have then, that $\dot{K} = \lambda(K(t)-\bar{K})$, or

$$\lambda(K(t)-\bar{K}) = -(C(t)-\bar{C}) + (\theta-n)(K(t)-\bar{K}) + \omega.$$

Differentiating (and using the fact that $\frac{d\bar{C}}{d\eta} = (\theta-n)\frac{d\bar{K}}{d\eta}$), we have

(2) $$\frac{dC(t)}{d\eta} = \lambda\frac{d\bar{K}}{d\eta} - (K(t)-\bar{K})\frac{d\lambda}{d\eta}.$$

[4] This is the path that satisfies the conditions: $\lim_{t\to\infty} K(t) \geq 0$, and $\lim_{t\to\infty} K(t)e^{-(\theta-n)t} \geq 0$ (which comes from integrating the equation of motion for the aggregate capital stock and imposing a boundary condition). These are necessary for a feasible infinite horizon consumption stream.

We need to evaluate $\frac{d\lambda}{d\eta}$. The characteristic polynomial for the system given by equation (1) is:

$$P = (\theta\gamma-\delta-\lambda)(\theta-n-\lambda)(\theta\gamma-n-\lambda) - \delta n(\theta\gamma-n-\lambda) - \delta n\theta(1-\gamma)(1-\eta).$$

Setting P equal to zero and differentiating allows us to calculate $\frac{d\lambda}{d\eta}$, evaluating the derivative at $\eta=1$. (Note that when $\eta=1$, $(\theta\gamma-\delta-\lambda)(\theta-n-\lambda) = \delta n$.)

$$\frac{d\lambda}{d\eta} = \frac{\delta n\theta(1-\gamma)}{(\theta\gamma-n-\lambda)[\theta-n+\theta\gamma-\delta-2\lambda]} > 0.$$

The sign of this derivative is the positive. From investigation of the polynomial P one can determine that $\lambda < \theta-\delta$, so recalling our assumption that $\theta\gamma-n-\lambda > 0$, the denominator is the product of two positive numbers.

We can see from equation (2) that the largest $\frac{dC(t)}{d\eta}$ can be is when $K(t) = 0$, because $\frac{d\lambda}{d\eta} > 0$. This is true in both the convergent and divergent cases. When $K(t) = 0$ (and evaluating the derivative at $\eta=1$):

$$\frac{dC(t)}{d\eta} = \frac{\delta n\theta(1-\gamma)(\theta\gamma-\delta)\omega}{\theta(\delta+n\gamma-\theta\gamma)}\left[\frac{\lambda}{\theta(\delta+n\gamma-\theta\gamma)(\theta\gamma-n)} + \frac{1}{(\theta\gamma-n-\lambda)(\theta-n+\theta\gamma-\delta-2\lambda)}\right]$$

$$= \frac{-\delta n\theta(1-\gamma)(\theta\gamma-\delta)\omega}{\pi}\left[\frac{\pi+\theta\gamma-n-\lambda}{\pi(\theta\gamma-n)(\theta\gamma-n-\lambda)(\pi-\lambda)}\right] < 0.$$

In this expression, $\pi > 0$ is the largest eigenvalue of the 2x2 system when $\eta = 1$. The largest value of $\frac{dC(t)}{d\eta}$ is negative.

So, when more rents accrue to equity owners (η falls below 1), currently living generations are able to capture more of the rents. The value of all future rents that will be paid to owners of equities are capitalized in the current price of equities. The wealth of the current generations is higher when equity holders receive some rents, and their consumption will be larger. Comparing this economy to a benchmark economy with the same initial capital stock (therefore the same output), an increase in the share of rents accruing to equity holders lowers saving.

In the stable economy capital accumulation will be lower, and the long-run levels of the capital stock and consumption will be lower. The initial consumption is higher in the economy in which some rents accrue to equity holders, but eventually aggregate consumption falls below what it would be in the economy with $\eta=1$.

In the unstable economy, growth will proceed at a slower rate in the economy in which η is lower (rents to labor are lower) because the per capita capital stock and consumption are diverging from steady state at a rate λ, which rises with η. Initial consumption is higher, but again, it must eventually fall below that of the economy with $\eta=1$.

We also have that $\dot{q} = \lambda(q(t)-\bar{q})$, so that

$$\lambda(q(t)-\bar{q}) = (\theta\gamma-n)(q(t)-\bar{q}) - \theta(1-\gamma)(1-\eta)(K(t)-\bar{K}).$$

Totally differentiating, we have:

$$(\lambda+n-\theta\gamma)\left[\frac{dq(t)}{d\eta} - \frac{d\bar{q}}{d\eta}\right] = \theta(1-\gamma)(K(t)-\bar{K}) + \theta(1-\gamma)(1-\eta)\frac{d\bar{K}}{d\eta} - (q(t)-\bar{q})\frac{d\lambda}{d\eta}.$$

Evaluating the change in q at $\eta=1$, the last two terms in the previous equation drop out, so that we have

$$\frac{dq(t)}{d\eta} = \frac{d\bar{q}}{d\eta} - \frac{\theta(1-\gamma)}{\theta\gamma-n-\lambda}(K(t)-\bar{K}). \tag{3}$$

This equation implies that $\frac{dq(t)}{d\eta}$ achieves its largest value when K(t) equals zero, whether the economy is stable or not. In this case (evaluated at $\eta=1$)

$$\frac{dq(t)}{d\eta} = \frac{-\theta(\theta\gamma-\delta)(1-\gamma)\omega}{\pi(\theta\gamma-n)(\theta\gamma-n-\lambda)} < 0.$$

So, as η falls slightly below 1, q rises. In the benchmark economy, q = 0, because there is no return to holding equities. Hence, in the comparison economy, q will be higher than in the benchmark economy.

To summarize both the convergent and divergent cases, when more rents accrue to firm owners, non-human wealth of current generations (K+q) is greater, because q is greater. Therefore, current consumption will be higher. Since the distribution of rents does not affect aggregate output ($= \theta\tilde{K} + \omega\tilde{L}$), saving will be lower as equity's share of rents rise. Hence, capital accumulation and growth will be smaller.

Thus, the growth rate of the economy depends on how rents are distributed. In the stable case, the steady state level of capital and consumption will be lower when labor's share of rents is smaller, while in the unstable case the permanent growth rate will be smaller.

B. Effects of the Externality on Growth

Now we will discuss the effects of the externality itself on growth. This model is not very different from many other models with investment externalities, except for the fact that the economy moves along a saddle path which could be either convergent or divergent.

We will consider the effects of small externalities. So, we will compare an economy in which there is no externality (γ=1) to one in which γ is slightly smaller than one. In our economy, there are constant returns to scale in the aggregate, no matter what the value of γ. A decrease in γ

represents not only a greater externality, but a lower private marginal product of capital as well. We are comparing economies that have the same aggregate technology and the same initial capital stock. We ask how differently the economies behave when externalities are relatively more important.

From the expressions for the steady state, (and evaluating the derivatives at $\gamma=1$), we have:

$$\frac{d\bar{K}}{d\gamma} = \frac{\omega\delta n}{\theta(\delta+n-\theta)^2} + \frac{(1-\eta)\delta n(\theta-\delta)\omega}{\theta(\theta-n)(\delta+n-\theta)^2} > 0.$$

$$\frac{d\bar{C}}{d\gamma} = (\theta-n)\frac{d\bar{K}}{d\gamma} > 0.$$

$$\frac{d\bar{q}}{d\gamma} = \frac{-(\theta-\delta)(1-\eta)\omega}{(\delta+n-\theta)(\theta-n)}.$$

When $\gamma = 1$, the roots are θ, $\theta-n-\delta$ and $\theta-n$. In the stable case, $\theta-n-\delta < 0$, and $\frac{d\bar{q}}{d\gamma} < 0$. In the unstable case, $\frac{d\bar{q}}{d\gamma} > 0$.

Analogous to our derivation of equation (2) is the derivation of

$$\frac{dC(t)}{d\gamma} = \lambda\frac{d\bar{K}}{d\gamma} - (K(t)-\bar{K})\frac{d\lambda}{d\gamma}.$$

Differentiating the characteristic polynomial, and evaluating the derivative for $\gamma = 1$ we get:

$$\frac{d\lambda}{d\gamma} = \frac{\theta(\delta+n(1-\eta))}{\delta+n} > 0.$$

The largest that $\frac{dC(t)}{d\gamma}$ can be when $\gamma=1$ is when $K(t) = 0$. This is true irrespective of the sign of $\lambda = \theta-n-\delta$. When $K(t) = 0$, we have

$$\frac{dC(t)}{d\gamma} = -\left[\frac{\omega\delta(n+\theta)}{\delta+n} + \frac{\omega(\theta+\delta)n(\theta-\delta)(1-\eta)}{\theta(\theta-n)(\delta+n)}\right] < 0.$$

Since $\frac{dC(t)}{d\gamma}$ is negative at its maximum, it is always negative.

So, when the private marginal product of capital is lower, current consumption is higher. Saving is lower (because the change in γ does not affect the aggregate level of output), and capital accumulation is lower. The stable economy converges to a smaller capital stock and consumption level, while the unstable economy grows continuously at a lower rate.

By following the same steps to derive equation (3), we can get

$$\frac{dq(t)}{d\gamma} = \frac{d\bar{q}}{d\gamma} - \frac{\theta(1-\eta)}{\delta}(K(t)-\bar{K}).$$

This derivative is largest in either the convergent or divergent case when $K(t) = 0$. In that case, $\frac{dq(t)}{d\gamma} = 0$. When $K(t) > 0$, $\frac{dq(t)}{d\gamma} > 0$, except when $\eta = 1$ (in which case, $\frac{dq(t)}{d\gamma} = 0$).

Hence, a greater externality must increase q if $\eta < 1$. That is simply because the greater the externality, the higher the rents earned by equity holders.

III. Fiscal Policies

The Pigouvian subsidy to capital sets $\rho = \theta(1-\gamma)$. This subsidy makes the rental rate on capital, r, equal to the social opportunity cost of capital, θ. When the Pigouvian subsidy is in place, the economy is at a Pareto optimum.[5]

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[5] In this overlapping generations model, a dynamically inefficient equilibrium is not possible because of the linear technology.

A. Taxes and Deficits

The dynamic effects of a subsidy depend upon the exact way that the government finances the subsidy. Options for the government include borrowing from the public, or raising non-distortionary taxes. There are two types of non-distortionary taxes available to the government: lump-sum taxes on consumers, or taxes on rents of the firms.

We will consider the following policy for non-distortionary taxes:

$$T(t) = \beta K(t) + Z,$$

where Z is constant. It is important to emphasize that even though the total amount of taxes collected (or subsidies paid, if T(t) is negative) depends on the aggregate capital stock, these taxes are levied as lump-sum taxes.

With this tax policy, the dynamic path for debt is given by:

$$\dot{D}(t) = (\rho+\gamma\theta-n)D(t) + (\rho-\beta)K(t) - Z.$$

We will also assume that the amount of taxes levied on rents of firms, P(t), is given by:

$$P(t) = \alpha K(t) + \varepsilon.$$

where ε is a constant. Recalling that $T(t) = P(t) + \tau(t)$, the amount of lump-sum taxes levied on consumers, $\tau(t)$, is given by:

$$\tau(t) = (\beta-\alpha)K(t) + Z-\varepsilon.$$

We now have that the value of equities evolves according to:

$$\dot{q}(t) = (\rho+\gamma\theta-n)q(t) + (\alpha-\theta(1-\gamma)(1-\eta))K(t) + \varepsilon.$$

Define "financial wealth", F(t), by $F(t) \equiv D(t) + q(t)$. We have:

$$\dot{F}(t) = (\rho+\gamma\theta-n)F(t) + (\rho+\alpha-\beta-\theta(1-\gamma)(1-\eta))K(t) + \varepsilon-Z.$$

We have chosen the policies for non-distortionary taxes in a way that the dynamics of the economy can be described with three state variables and three dynamic equations. As we shall see, this set of fiscal policies is special in the sense that it allows the government to arrive at any desired distribution of the external rents.

The dynamic system for the economy is:

$$(4)\qquad \begin{bmatrix} \dot{C}(t) \\ \dot{K}(t) \\ \dot{F}(t) \end{bmatrix} = \begin{bmatrix} \rho+\theta\gamma-\delta & -\delta n & -\delta n \\ -1 & \theta-n & 0 \\ 0 & \rho+\alpha-\beta-\theta(1-\gamma)(1-\eta) & \rho+\theta\gamma-n \end{bmatrix} \begin{bmatrix} C(t) \\ K(t) \\ F(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \\ \varepsilon-Z \end{bmatrix}.$$

We can now make the following observation about the economy when the Pigouvian tax is imposed.

<u>Observation 1</u>: When the Pigouvian tax, $\rho = \theta(1-\gamma)$, is imposed, and when $\rho+\alpha-\beta-\theta(1-\gamma)(1-\eta) = 0$ and $\varepsilon-Z = 0$, the path for aggregate consumption and capital will be the same as the laissez-faire economy with the same aggregate production function and no externality (γ=1).

Under the conditions stated in the observation, $F(t) = 0$ for all t. Although $q(t)$ and $D(t)$ need not be zero, the paths of $C(t)$ and $K(t)$ are independent of q and D, and are determined by:

$$\dot{C}(t) = (\theta-\delta)C(t) - \delta nK(t)$$

$$\dot{K}(t) = -C(t) + (\theta-n)K(t) + \omega.$$

These are also the dynamic equations for the economy with no externality (so the aggregate production function is given by $\theta\tilde{K}(t) + \omega\tilde{L}(t)$, which is identical to the production function of the representative firm), and no government intervention.

We can also see from examination of equation (4) that the equations for the evolution of consumption and the capital stock are not directly affected by the share of rents going to labor, η, the parameter governing the behavior of total non-distortionary taxes, β, or the parameter determining the share of non-distortionary taxes that are levied on rents paid to firms, α. We can therefore state:

Observation 2: A larger value of α or a smaller value of β has the exact same effect on the path for the economy as a smaller value of η.

This follows from examination of equation (4), noting that α, β and η appear only as coefficients on K(t) in the equation for $\dot{F}(t)$. A larger α or a smaller β leads to the same behavior as a larger η. Because C, K and F are state variables for this system, the specified changes in α, β and η have the same effect on all variables in the economy.

The intuition of this result is as follows. A greater value of α simply implies that the rents for equity holders are being taxed at a greater rate. This has the same effect as if the external rents accruing to owners of equities in the laissez-faire economy were lower.

A greater value of β implies that taxes are being shifted more toward future generations. The current debt of the government must be lower with the government's solvency constraint imposed. Since future generations bear more of the tax burden, the currently alive are better off, just as if they were receiving a greater share of external rents.

Tax collection policies that depend upon the aggregate capital stock allow us to change the parameters the fiscal policies used, to mimic the distributional effects of the external rents. In contrast, changes in ε and Z cannot completely offset the effects of a change in η on the dynamic system.

B. Dynamic Effects of Budgetary Policies

In this section, we will examine the effects of higher tax collections and of shifting more of the tax burden on to owners of firms.

We will assume the Pigouvian subsidy is in place, so that $\rho = \theta(1-\gamma)$. All changes in lump-sum tax policies will not effect efficiency, only intertemporal distribution.

We will also assume that ε and Z are zero, so that total non-distortionary taxes are given by $T(t) = \beta K(t)$, and the amount of taxes imposed on rents of firms is $P(t) = \alpha K(t)$. We assume that $\alpha \le \theta(1-\gamma)(1-\eta)$, so that after-tax rents are never negative.

We will compare two economies with different tax policies. Define $\Gamma = \alpha-\beta+\theta\eta(1-\gamma)$. In the benchmark economy, $\Gamma = 0$. In the comparison economy, Γ is slightly less than 0, either because the economy collects more taxes in total (higher β), or because the burden of taxes falls less heavily on owners of firms (smaller α).

The dynamic system for the economy can be represented simply as:

$$\begin{bmatrix} \dot{C}(t) \\ \dot{K}(t) \\ \dot{F}(t) \end{bmatrix} = \begin{bmatrix} \theta-\delta & -\delta n & -\delta n \\ -1 & \theta-n & 0 \\ 0 & \Gamma & \theta-n \end{bmatrix} \begin{bmatrix} C(t) \\ K(t) \\ F(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \\ 0 \end{bmatrix}.$$

The steady-state for the economy is given by:

$$\bar{K} = \frac{(\theta-\delta)(\theta-n)\omega}{\theta(\delta+n-\theta)(\theta-n)-\Gamma\delta n}.$$

$$\bar{C} = \frac{[\delta n(\theta-n)-\Gamma\delta n]\omega}{\theta(\delta+n-\theta)(\theta-n)-\Gamma\delta n}.$$

$$\bar{F} = \frac{-\Gamma(\theta-\delta)\omega}{\theta(\delta+n-\theta)(\theta-n)-\Gamma\delta n}.$$

We will consider how a change in Γ affects the dynamic path of the economy. That will allow us to compare the benchmark economy and the comparison economy, assuming they have the same initial capital stock but different tax policy parameters, α and β.

The effect of a change in Γ on the steady state (evaluating the derivatives at $\Gamma = 0$) is given by:

$$\frac{d\bar{K}}{d\Gamma} = \frac{\delta n(\theta-\delta)(\theta-n)\omega}{[\theta(\delta+n-\theta)(\theta-n)]^2}.$$

$$\frac{d\bar{C}}{d\Gamma} = (\theta-n)\frac{d\bar{K}}{d\Gamma}.$$

$$\frac{d\bar{q}}{d\eta} = \frac{-(\theta-\delta)\omega}{[\theta(\delta+n-\theta)(\theta-n)]^2}.$$

Analogous to equation (2) above, we have:

$$\frac{dC(t)}{d\Gamma} = \lambda\frac{d\bar{K}}{d\Gamma} - (K(t)-\bar{K})\frac{d\lambda}{d\Gamma}.$$

To evaluate this expression for $\frac{dC(t)}{d\Gamma}$, we calculate $\frac{d\lambda}{d\Gamma}$ evaluated at $\Gamma = 0$ (when $\Gamma = 0$, $\lambda = \theta-\delta-n.$), to get:

$$\frac{d\lambda}{d\Gamma} = \frac{n}{\delta+n} > 0.$$

Because $\frac{d\lambda}{d\Gamma} > 0$, $\frac{dC(t)}{d\Gamma}$ is largest when $K(t)$ is zero -- in either the convergent or divergent case. Then,

$$\frac{dC(t)}{d\Gamma} = \frac{-(\delta+\theta)(\theta-\delta)n\omega}{(\delta+n)\theta^2(\theta-n)} < 0.$$

So, even at its largest, $\frac{dC(t)}{d\Gamma}$ is negative.

We also have, following the derivation of (3), that

$$\frac{dF(t)}{d\Gamma} = \frac{d\bar{F}}{d\Gamma} - \frac{1}{\delta}(K(t)-\bar{K}).$$

This derivative is largest when $K(t) = 0$. In this case

$$\frac{dF(t)}{d\Gamma} = \frac{-(\theta-\delta)\omega}{\delta\theta(\theta-n)} < 0.$$

We conclude that in the comparison economy with a lower value of Γ, initial consumption and the initial value of financial assets are greater than in the benchmark economy.

A lower value of Γ implies that a higher share of the tax burden will fall on future generations, either because β is greater or α is lower. Thus, the current generations are wealthier, which means that their consumption is higher. Saving in the economy will be lower. In the stable case, the economy with the lower value of Γ will converge to a lower capital stock, and a lower steady-state consumption. In the unstable case, the rate of growth of per capita income, λ, will be lower.

If we let the benchmark economy be one where both debt and the value of firms is zero (so $\beta = \theta(1-\gamma)$ and $\alpha = \theta(1-\gamma)(1-\eta)$), then if the benchmark economy had a higher total tax rate (a higher β), it would also have a greater initial debt. So, if we compare two economies with the same capital stock at time t, the one that has been running larger budget deficits and has a higher government debt will have greater current consumption. However, in the stable case its long-run capital stock and consumption will be lower, and in both cases, its growth rate will be lower.

Interestingly, an economy which places a smaller share of the tax burden on firm-owners, (a smaller α) will replicate the consumption and capital accumulation paths of the economy with higher initial debt and higher β. In this case, the lower value of α will increase the value of firms, thus encouraging current consumption and discouraging saving.

Neither of the fiscal policies discussed in this section are distortionary. They have no direct effects on factor supplies or factor demands, nor do they alter current output. Their effects are distributional. But, by changing the distribution of income across generations, they change the growth path.

IV. Conclusions

We have seen that the growth path of the economy depends on the size of the externalities generated by capital and labor. This is not surprising. It is the basis for much of the analysis of the new growth theory.The emphasis of this paper has been on the effects of the distribution of the rents generated by the externality.

We have constructed an extremely simple example economy with an investment externality typical of those in the new growth literature. We see that as a lower fraction of the rents are captured by workers, and more by firm-owners, that current consumption will be greater, and saving will be lower. The growth rate will be lower and long-run output and consumption are lower.

In the economy we study, a policy of subsidizing capital will lead to a Pareto improvement in welfare. But, the government may also be interested in distributional policies which allow it to increase the value of social welfare. The fiscal policies that are chosen to achieve some desired intergenerational distribution of income or growth rate should depend on the distribution of rents.

The empirical importance of the distribution effects is an open issue. It should be possible to detect differences in the growth rates of economies that have different rent distributions, but we leave that to future research.

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