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PRICE NORMALIZATION AND EQUILIBRIA IN
GENERAL EQUILIBRIUM MODELS OF INTERNATIONAL
TRADE UNDER IMPERFECT COMPETITION

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Abstract

Applied general equilibrium modeling has become a widely used tool in analyzing the effects of changes in tax and trade policies of developed and developing countries. Some of these models introduce non-competitive market structures into the analysis to capture market imperfections in real economies. However, in these applications the difficulties encountered by theorists for extending general equilibrium analysis to economies with monopolistic competition and oligopoly have been ignored. In particular, some of the applications (which depart from the large-group-free-entry-zero-profit variant of the Chamberlinian monopolistic competition are especially vulnerable to the critique that the results can be very sensitive to the arbitrary choice of price normalization. We illustrate how price normalization matters in a model representative of those used in the trade literature with oligopoly imply that the effects of policy changes on welfare and resource allocation are sensitive to the choice of numéraire in these models.

KEY WORDS: Applied General Equilibrium Modeling, Monopolistic Competition, Price Normalization
I. Introduction

Since the late 1970's, the most active area for publication in international economics has been the application of models of imperfectly competitive industries from the industrial organization literature to international trade theory and policy. Applications to positive economics emphasize the ability of increasing returns to scale with the associated imperfectly competitive producer behavior to explain intra-industry international trade. For normative economics, some of these models provided a rationale for active government intervention to promote national firms in oligopolistic international markets under certain conditions. Recently, these models have been applied to the quantitative analysis of trade policy changes in computable general equilibrium versions.

For some time, general equilibrium theorists have recognized major difficulties introducing oligopoly and monopolistic competition into general equilibrium analysis. Although these problems are well-known among theoreticians, their implications for the theoretical and quantitative analysis of international trade policies under imperfect competition has not been acknowledged in the trade literature. The purpose of this paper is to point out the potential importance of these pitfalls for the analysis of trade with imperfect competition.

Difficulties that arise for the theory of monopolistic competition and oligopoly in general equilibrium originate in the optimization problem for non-price-taking firms. When a monopolistic or oligopolistic firm takes account of the income effect generated by its profit on the demand for its output, convexity can fail for both the choice set and the objective function for the profit-maximizing firm, and existence of equilibrium
cannot be proved in general. The possibility of non-existence of an
equilibrium in which some firms behave in Cournot fashion and other agents
act as price-takers (defined as a Cournot-Walras equilibrium by Gabszewicz
and Vial [1972]) has been shown in examples by Dierker and Grodal [1986]
and proven to be a generic property by Bohm [1990].

It is also known that the set of equilibria is sensitive to the manner
in which relative prices are normalized. That is, the equilibrium
allocation and vector of relative prices change with the price
normalization, and equilibrium may fail to exist for some choices of
normalization and not for others. This is a consequence of the assumption
that firms maximize profits and take account of the effect their decisions
have on market prices. Typical partial equilibrium models of oligopoly set
the nominal wage equal to one, implicitly assuming that labor serves as the
numéraire. The firm then maximizes the purchasing power of profits in
units of labor. If the firm’s product is chosen as numéraire, then the
oligopolist would maximize the purchasing power of profits in terms of that
good. Changing the numéraire has no effect on Walrasian equilibria since
the objective function for profit-maximizing firms is unaffected if all
prices are taken as given. Changing price normalization typically leads
price-setting firms to choose different production plans.

Two types of models have gained popularity in the literature on
international trade under imperfect competition. Models of one type are
general equilibrium versions of the Chamberlin model of monopolistic
competition for a large group of firms with free entry. Krugman [1979,
1980, 1981], Dixit and Norman [1980], Ethier [1979] and several others
adopt the specific general equilibrium model of Chamberlinian monopolistic
competition of Dixit and Stiglitz [1977], while Helpman [1981] and
Lancaster [1980] use a variant based on Lancastrian model of demand for product characteristics. Models of the other type are based on partial equilibrium models of oligopoly. The simple duopoly model presented in the critical analysis by Eaton and Grossman [1986] represents this literature well.

We show how the choice of price normalization affects equilibrium relative prices and allocations in a simple general equilibrium model in the spirit of the Dixit-Stiglitz and Eaton-Grossman models. Production technologies display constant returns to scale in one case and increasing returns to scale in the other. Firms behave as Cournot oligopolists while households act as price-takers. We argue that the dependence of equilibria on numéraire choice is an important problem for models of international trade under imperfect competition.

Once it is established that equilibria are sensitive to the specification of the numéraire, it is clear that estimates of the effects on welfare and resource allocation of changes in indirect or direct tax rates, tariff rates or quantitative restraints on international or national trade from computable general equilibrium models incorporating imperfect competition should be treated with caution. Because imperfect competition is included in these models, policy reforms take place in a second-best world. Since trade liberalization or reduction of a distortionary tax, in general, has an ambiguous effect on welfare in all second-best environments, simulation using a computable model based on estimated equations is a useful tool for resolving the ambiguity and assessing the impact of trade and fiscal reforms under such market structures. The analyses of trade reforms using computable general equilibrium with monopolistically competitive or oligopolistic industries by Harris [1984a,b],
Cox and Harris [1985], de Melo and Roland-Holst [1990a,b] and Devarajan and Rodrik [1989a,b], among others, are all subject to the criticism that the results could be sensitive to the arbitrary choice of price normalization made. Changing the numéraire in a computable general equilibrium version can affect the size and even sign of the effects of policy changes on welfare and allocation of factors across industries.

II. Profit Maximization and Numéraire Choice

Consider an economy in which there are at least two price-setting firms. There are $n$ goods and a single price-taking representative household. The production function for each produced good is continuously differentiable and is given by $f_j(x_1, x_2, \ldots, x_n)$. The vector of absolute (not normalized) prices is denoted by $(p_1, p_2, \ldots, p_n)$. The profit for a price-setting firm producing good $j$ in absolute prices is given by

$$\Pi_j = p_j f_j(x_1, \ldots, x_n) - \sum_{i=1}^{n} p_i x_i.$$ 

If good 1 is chosen as the numéraire, the firm's profit function is given by

$$\tau_j = \frac{p_j}{p_1} f_j(x_1, \ldots, x_n) - \sum_{i=1}^{n} \frac{p_i}{p_1} x_i.$$ 

Alternatively, if good $n$ is taken to be the numéraire, the firm's profit function becomes

$$\tau_j = \frac{p_j}{p_n} f_j(x_1, \ldots, x_n) - \sum_{i=1}^{n} \frac{p_i}{p_n} x_i.$$
so that

\[ \hat{\tau}_j = \frac{P_j}{p_n} \tau_j. \]

Hence, if \( P_1/p_n \) is taken as given by the firm, then maximization of \( \tau_j \) and \( \hat{\tau}_j \) would be equivalent. If \( P_1/p_n \) is not taken as given, then the first-order conditions for profit maximization by firm \( j \) under these two choices of numéraire are related by the equation

\[ \frac{\partial \hat{\tau}_j}{\partial x_j} = \frac{P_1}{p_n} \frac{\partial \tau_j}{\partial x_j} + \tau_j \frac{\partial (P_1/p_n)}{\partial x_j}, \quad \text{for each } j = 1, \ldots, n, \]

where \( \frac{\partial (P_1/p_n)}{\partial x_j} \) is firm \( j \)'s conjecture of the effect of an increase in firm \( j \)'s output on the relative price, \( P_1/p_n \).

Note, that if \( \tau_j = \hat{\tau}_j = 0 \), then \( \frac{\partial \hat{\tau}_j}{\partial x_j} = \frac{P_1}{p_n} \frac{\partial \tau_j}{\partial x_j} \) so that \( \frac{\partial \hat{\tau}_j}{\partial x_j} \leq 0 \) if and only if \( \frac{\partial \tau_j}{\partial x_j} \leq 0 \).

This implies that the necessary conditions for profit maximization by firm \( j \) are identical under these two choices of relative price normalization only if the profit expressed in terms of either numéraire is zero in the particular equilibrium.

In general, the objective function and supply correspondence for a profit-maximizing oligopolist depend upon the normalization chosen for relative prices. Whenever the profit of some non-price-taking firm is not zero in equilibrium, changing the numéraire changes equilibrium relative prices and allocations. If the profits of all firms are zero in a
particular equilibrium for one choice of normalization for relative prices, then that equilibrium will survive a change in the method of normalizing prices.

Below we assume that the firm recognizes the income effect of its own profit on demand, and we restrict our attention to Cournot conjectures for simplicity. In the literature on monopolistic competition and oligopoly in general equilibrium, this is called the objective demand approach (see Hart [1985]). Under Cournot conjectures, each firm includes the effect of its output level on profit income in its calculation of the effect of its decision on demand.

III. Some examples

A simple example economy is used to demonstrate the importance of numéraire choice for models of international trade and policy under monopolistic competition and oligopoly. This model is a simple general equilibrium version of that used by Eaton and Grossman [1986] to show the dependence of optimal national policies on the conjectures assumed to be held by firms. It also is recognizably close to the Dixit-Stiglitz model for a small group of firms.

There are three commodities and two firms. There is a single representative household. For the sake of simplicity, we do not identify countries and discuss international trade explicitly. It is straightforward to envision that one firm operates in each country and that households have identical tastes across the border.

The endowment of the representative household consists of a supply of only one commodity, leisure, denoted $z$. Production of each of the other
two goods, x and y, requires an input of labor. The amount of labor needed to produce one unit of good x is $a_1 \cdot x$, and the labor requirement per unit of good y is $a_2 \cdot y$. The household has an initial endowment of leisure equal to unity. The preferences for the household display constant elasticity of substitution and are represented by the utility function:

$$U = x^\theta + y^\theta + z^\theta,$$

for $0 < \theta < 1$.

The household behaves as a perfect competitor choosing how much of goods x and y to purchase and how much labor to supply. Each of the two firms behaves as a Cournot duopolist, taking into account how its production decision affects equilibrium prices under the assumption that the other firm's output is fixed. That is, the firm does not take the wage rate, as well as the price of its output, as given.

Maximization of utility subject to the household's budget yields the following set of first-order conditions:

$$\frac{1-\theta}{x} = \frac{p_1}{w},$$

$$\frac{1-\theta}{y} = \frac{p_2}{w},$$

$$w + \Pi_1 + \Pi_2 = p_1 \cdot x + p_2 \cdot y + w \cdot z,$$

where $p_1$, $p_2$ and w are the absolute prices of goods x, y and z, respectively. $\Pi_1$ and $\Pi_2$ are the profits in terms of absolute prices for firm 1 and firm 2, respectively.

If leisure is chosen as the numéraire, then the profits for the two firms are given by:
\[ \tau_1 = \left( -\frac{x}{z} \right)^{1-\theta} x - (a_1 + \beta_1 x), \text{ and} \]
\[ \tau_2 = \left( -\frac{x}{y} \right)^{1-\theta} y - (a_2 + \beta_2 y), \]

where \( z = 1 - (a_1 + \beta_1 x) - (a_2 + \beta_2 y) \).

Note that the inverse demand function for good \( x \) is
\[ \frac{1 - (a_1 + \beta_1 x) - (a_2 + \beta_2 y)}{x} \cdot (\theta - 1). \]

The first-order conditions for profit maximization under the Cournot assumption subject to the equilibrium condition are

\[ \frac{\partial \tau_1}{\partial x} = \theta q_1 - \beta_1 (1 - \theta) q_1 \theta/(\theta - 1) - \beta_1 \leq 0, \text{ and} \]
\[ \frac{\partial \tau_2}{\partial y} = \theta q_2 - \beta_2 (1 - \theta) q_2 \theta/(\theta - 1) - \beta_2 \leq 0, \]

where \( q_1 \equiv \frac{p_1}{w} \) and \( q_2 \equiv \frac{p_2}{w} \) are the relative prices of goods \( x \) and \( y \) in terms of leisure, respectively.

If good \( x \) is chosen as the numéraire, then the profit for each firm is given by:

\[ \hat{\tau}_1 = x - \left( -\frac{x}{z} \right)^{1-\theta} (a_1 + \beta_1 x), \text{ and} \]
\[ \hat{\tau}_2 = \left( \frac{x}{y} \right)^{1-\theta} y - \left( -\frac{x}{z} \right)^{1-\theta} (a_2 + \beta_2 y). \]

The first-order condition for a solution to firm 1's profit-maximization problem is
(4') \[ \frac{\partial \hat{r}_1}{\partial x} = 1 - \left[ \frac{a_1}{z} + \beta_1 \cdot q_1^{1/(\theta-1)} \right] q_1^{\theta/(1-\theta)} (1 - \theta) \left[ 1 + \beta_1 \cdot q_1^{1/(\theta-1)} \right] - \beta_1 \cdot q_1^{-1} \leq 0. \]

The first-order condition for a solution for firm 2 is

(5') \[ \frac{\partial \hat{r}_2}{\partial y} = q_1^{-1} \left[ \theta q_2 - \beta_2 (1 - \theta) \left( \frac{a_2}{z} + \beta_2 \cdot q_2^{1/(\theta-1)} \right) - \beta_2 \right] \leq 0. \]

A Cournot-Walras equilibrium when leisure is the numéraire is found by solving inequalities 1, 2, 4, 5 and

(6) \[ z \leq 1 - (a_1 + \beta_1 x) - (a_2 + \beta_2 y). \]

When good \( x \) is chosen to be the numéraire, then the general equilibrium relative prices and allocation solve inequalities 1, 2, 4', 5' and 6.

Clearly, these two sets of necessary conditions differ in general. As noted by Dierker and Grodal [1986] and Hart [1985], Cournot-Walras equilibria may not exist for some price normalization rules.

We give the following numerical example to illustrate that the sets of equilibria can differ when neither is non-empty.

Example 1:

Let \( a_1 = a_2 = 0 \) and \( \beta_1 = \beta_2 = 1/3 \), and let \( \theta = 0.5 \). When leisure is chosen to be the numéraire, there is a single equilibrium. This is

\[ q_1 = q_2 = 1, \] and

\[ x = y = z = 3/5 = 0.6. \]

This allocation yields utility,
\[ U = 3 \left(\frac{3}{5}\right)^{0.5} \approx 2.3228. \]

If good x is chosen as the numéraire, then the solution to the first-order conditions that satisfies the second-order conditions for profit-maximization of the Cournot-duopolists is

\[ q_1 = 0.6369, \]
\[ q_2 = 0.8285, \]
\[ x = 1.0684, \]
\[ y = 0.6314, \text{ and} \]
\[ z = 0.4334. \]

This allocation yields utility,
\[ U = 2.4866. \]

The unique Pareto optimum (the Walrasian equilibrium allocation) is given by

\[ x = y = \frac{9}{7}, \]
\[ z = \frac{1}{7}. \]

This allocation gives the household utility,
\[ U = 7^{0.5} \approx 2.6458. \]

Example 2:

As a second example, we allow fixed costs to be positive. Keeping the rest of the example the same, let \( q_1 = q_2 = 0.2 \). The solution for the unique Cournot-Walras equilibrium when leisure is the numéraire is

\[ q_1 = q_2 = 1, \text{ and} \]
\[ x = y = z = \frac{9}{25} = 0.36. \]

Utility for the household is equal to
\[ U = 1.8. \]

When good x is chosen to be the numéraire, the equilibrium is

\[ q_1 = 0.9050, \]
\[ q_2 = 0.9779, \]
\[ x = 0.4173, \]
\[ y = 0.3574, \]
\[ z = 0.3418. \]

The equilibrium level of utility for the household and profits for each firm are given by

\[ U = 1.8284, \]
\[ \hat{\tau}_1 = 0.3450, \]
\[ \hat{\tau}_2 = 0.0336. \]

The unique Pareto optimal (and Walrasian equilibrium) allocation is

\[ x = y = 27/35, \]
\[ z = 3/35, \]
resulting in a level of utility of \[ U = 2.0494. \]

In these examples, we can allow free entry and exit of firms in the manner of Dixit and Stiglitz [1977]. The utility function is given by

\[ U = \theta + \sum_{i=1}^{n} \theta x_i, \]

where \( n \), the number of product types, is determined by free entry. Firms enter until the addition of one more firm results in negative equilibrium profits for that firm. Applications of models of monopolistic competition to international trade assume that the profits of every firm are zero in condition of equilibrium (for example, Krugman [1979, 1980 and 1981], Helpman [1982] and Helpman and Krugman [1985]). When zero profits for all firms is imposed as a condition of equilibrium, the particular price normalization rule chosen has no effect on any firm's decision problem so that the set of equilibrium relative prices and allocations are the same for all such rules. However, free entry is not the same as assuming that profits are zero for every active firm in equilibrium when technologies
display increasing returns to scale. It is only true in exceptional cases that imposition of the zero profit condition along with the standard equilibrium conditions leads to an equilibrium with an integer number of firms in these models.

In the second example, it is easily checked that entry by a third firm is not profitable. The addition of another Cournot oligopolist producing either a new product or one of goods x or y leads to an equilibrium in which at least one firm earns negative profit for both numéraire choices.

IV. Alternative Objectives for the Firm

In a Walrasian equilibrium, price normalization does not affect equilibrium relative prices and allocations. The supply correspondence for a profit-maximizing firm that takes relative prices as given does not depend upon the numéraire choice. There is no difference between maximizing profit and maximizing the purchasing power of profit in terms of any commodity bundle. In Cournot-Walras equilibria the objective for the firm is to maximize profit in terms of the bundle of goods specified by the price normalization. When the firm recognizes its effect on prices, this objective varies with the numéraire.

As Marshall [1922] pointed out, profit maximization may not be the appropriate objective for imperfectly competitive firms, since owners care about what their profit can buy so that there is a tradeoff between monetary profit and the cost of the consumption bundle owners desire. When the firm can influence prices, it is natural to substitute utility maximization for the owners for profit maximization.

Suppose that there is a single owner for each of the two firms in our
examples. Replacing profit maximization with utility maximization for this single shareholder eliminates the dependence of the firm's optimal production plan on price normalization under the assumption that the shareholder behaves as a price-taker in her consumption decision. Thus, suppose that the owner of firm $j$ has preferences satisfying standard properties and that these yield the indirect utility function

$$V_j(p_1', p_2', \ldots, p_n', w, w_1^j).$$

(We assume that household $j$ owns all of firm $j$ and no shares in other firms for simplicity only.) Since this indirect utility is homogenous of degree zero in prices, the optimal production plan is independent of the numéraire. However, the assumption that the owner as a consumer ignores the influence she has on the market in her role as a producer is implausible. If this influence is recognized, then the problem is no longer simple.$^3$

Applications of the large group Chamberlinian monopolistic competition model (for example, Krugman [1979, 1980, 1981], Helpman [1981]) assume that $p_j$ has no direct effect on $V_j$. Under this assumption, utility maximization for the owner implies that the firm should maximize her income expressed in any price normalization. It does not matter how prices are normalized. In this case, maximizing profit with leisure as the numéraire is the same as maximizing indirect utility for household $j$. However, maximizing the profit of firm $j$ for other choices of numéraire is not identical to maximizing her income. We do not need to worry about the profit maximization assumption in the monopolistic competition models offered in the international trade literature because they impose zero profits as an equilibrium condition and assume that utility is broad-based in the sense that the first-order effect of a rise in the price of any commodity on
shareholder utilities is null.

Difficulties for modelling general equilibrium under imperfect competition are not easily solved by the substitution of shareholder utility for profit. Dierker and Grodal [1986] show that a Cournot-Walras equilibrium need not exist when there is a single shareholder with regular preferences over consumption bundles in an example economy in which there are two firms with strictly convex production sets. If there are multiple shareholders and they have different tastes, then the problem is how to aggregate these preferences into an objective function for the firm.

V. Conclusion

The difficulties encountered by theorists for extending general equilibrium analysis to economies with monopolistic competition and oligopoly have been ignored in the literature on international trade under imperfect competition. Under the large group assumptions made in the applications of the Dixit-Stiglitz model of Chamberlinian monopolistic competition to international trade, these problems do not arise. When these are relaxed the assumption of profit maximization becomes problematic. Utility maximization for the owners does not provide a thoroughly adequate remedy.

Computable general equilibrium models of trade with imperfect competition are especially vulnerable to the critique that the results can be very sensitive to the arbitrary choice of price normalization. Our examples of how price normalization matters in a model representative of those used in the trade literature with oligopoly imply that the effects of policy changes on welfare and resource allocation are sensitive to the
choice of numéraire in these models. The computable general equilibrium versions proposed by Harris [1984a,b], Cox and Harris [1985], Burniaux and Waelbroeck [1992], de Melo and Roland-Holst [1990a,b] and Devarajan and Rodrick [1989a,b] make a variety of assumptions about the nature of imperfect competition. These range from Chamberlinian monopolistic competition for a large group of firms with free entry to the behavior proposed by Eastman and Stykolt [1967] in which each imperfectly competitive domestic firm sets its price equal to the tariff-inclusive price of the competing import good. All these models are subject to the criticism that the estimated effects of trade reforms might depend on the price normalization chosen by the authors.
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Footnotes

1Ginsburgh [1990] shows the possibility that elimination of a consumption tax in a simple general equilibrium model with oligopoly can lead to an increase in utility for a representative household under one choice of numeraire and a decrease under another. He uses an input-output model with Leontief technology for two firms that violates the 'no free lunch' axiom, so that the technologies are specified with exogenous bounds on outputs.

2Alternatively, Smith and Venables [1988, 1989] present computable partial equilibrium industry models with imperfect competition. These follow the spirit of oligopoly models in the trade theory literature, such as Eaton and Grossman, in making an implicit assumption about the numeraire and an explicit one about firms not perceiving the income effect of their own profit on demand.

3This problem could be ignored by assuming that there are many owners of each firm who have identical homothetic preferences. In that case, it might be reasonable to assume that each owner acts as a price-taker in her consumption decision while the firm maximizes the representative owner's indirect utility function taking account of its influence on relative prices.