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FACTOR SUBSTITUTION POSSIBILITIES IN INDIAN MANUFACTURING INDUSTRIES

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Sharif Mohammad *

Relative factor prices and the extent of substitution possibilities among factors of production play an important role in formulating policies for generating more employment opportunities and thus redistributing income in the developing countries. The argument for developing small scale industries, in such countries, through encouraging these industries by providing cheap loans, raw materials, marketing facilities, etc., is generally justified on the ground that these units use labour intensive technology. In a labour-surplus (and short of capital) economy the most appropriate technology is one which reflects the relative supplies of these factors.

In the present exercise we attempted at estimating the substitution elasticities between labour and capital (two main factors of production) using a standard VES-production functions (which is a generalized variable elasticity of substitution function).

Elasticity of substitution (σ) may be defined as the measure of the ease with which the varying factor can be substituted for others; Hicks (1) It is defined as:

$$\sigma = - \frac{dx}{x} / \frac{dR}{R} \qquad ...(1)$$

where R is the marginal rate of substitution of labour for capital:

$$R = - \frac{dK}{dL} = \frac{\partial v/\partial L}{\partial v/\partial K} \qquad ...(2)$$

where V = value added, L = labour, and K = capital.

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If the production function is homogeneous of degree 1, σ can be written as:

$$\sigma = \frac{\partial v/\partial K \cdot \partial v/\partial L}{v.\partial^2 v/\partial K\partial L} \qquad \ldots(3)$$

Allen (2).

Substituting partial derivatives and cross-second derivatives into (3) we obtain; (according to Lu and Fletcher (3): Elasticity of substitution:

$$\sigma = \frac{b}{1-cf/xf'}$$

or

$$\sigma = \frac{b}{1-c(1+R/x)} = \frac{b}{1-c(1+\frac{wL}{rK})} \qquad \ldots(4)$$

where wL = share of labour in value added,

rK = share of capital in value added,

X is capital labour ratio and b and c are coefficients derived from the production function estimated and discussed below:

Starting with the following functions:

$$\log V/L = \log a + b \log W + c \log K/L + e \qquad \ldots(5)$$

when the production function V = F (K,L) is homogeneous of degree one, then V/L = F(K/L,1). Set V/L = Y and K/L = X, then we have Y = f(X) or V = L f(X). Let W be the wage rate with output as numeraire. Assuming competitive markets for labour as well for the product; Arrow, Chenery, Minhas, and Solow (4);

$$W = f(X) - xf'(X)$$
$$= Y-x\ (dy/dx), \text{ and} \qquad \ldots(6)$$
$$r = f'(X)$$

where $f'(X)$ is the marginal product of capital, $f(X) - f'(X)$ the marginal product of labour and r returns to capital. Now by substituting (6) into (5) we get:

$$\log Y = \log a + b \log (Y - X(dY/dX)) + c \log X \qquad \ldots(7)$$

By solving for dY/dX and substituting for $Z = Y^{1-\frac{1}{b}}$, Lu and Fletcher (5) derive the following function:

$$v = [\beta K^{-\rho} + \alpha\eta(K/L)^{-c(1+\rho)}L^{-\rho}]^{-\frac{1}{\rho}} \qquad \ldots(8)$$

where

$$\rho = \frac{1}{b-1},\ \eta = \frac{1-b}{1-b-c},\ \alpha = a^{-\frac{1}{b}},$$ and β is the constant of

integration. By setting $\alpha = (1-\delta)r^{-\rho}$ and $\beta = \delta r^{-\rho}$, we obtained:

$$v = r[\delta K^{-\rho} + (1+\delta)\eta(K/L)^{-c(1+\rho)}L^{-\rho}]^{-\frac{1}{\rho}} \qquad \ldots(9)$$

This function is a generalized form of CES production function. Lu(6) has shown the following properties of this function:

(1) Positive marginal products,

(2) Downward sloping marginal product curves over the relevant ranges of output,

(3) Homogeneity of degree one, and

(4) Variable elasticities of substitution.

Data and Estimation of Elasticities

We have used the Annual Survey of Industries (ASI)(7) data, for the year 1966. Out of more than 50 industries (or group of industries), given in the ASI Reports, we have estimated elasticities for 43, industries. In this cross section of data various States' information were the observations for each industry. For capital (K) we have taken only fixed capital into consideration. With this data we have estimated following equations to test different hypotheses about the form of production, returns to scale and elasticity for each industry: (8):

(10a) $\text{Log } V/L = \log a_1 + b_1 \log w + U_1$

(10b) $\text{Log } V/L = \log a_2 + b_2 \log w + C_2 \log K/L + u_2$

(10c) $\text{Log } V/L = \log a_3 + b_3 \log w + C_3 \log K/L + d_3 \log L + u_3$

where

V = value added in Rs.

L = Employment (total workers, i.e., production plus non-production workers)

w = Average wage rate of workers in Rs. per man-days.

K = total fixed capital in Rs.

The form of production function can easily be tested. For example:

(a) our null hypothesis may be $H_o: d_3 = 0$ tests whether the returns to scale are constant,

(b) $H_o : C_1 = o$ tests whether the production function is of CES or VES form

(c) Further $b_1 = o$ when $C_1 = o$, or that $b_1 = 1$ tests whether production function is of the Cobb-Douglas type; and

(d) That $b_1 = o$ when $C_1 = o$ tests whether the production function is of the fixed input coefficient (Leontief) type. (Cf, Yeung and Tsang (9)).

Results

Table I presents the details of the three equations (10a, 10b, and 10c)estimated for 43 ASI industry groups for the year 1966. The first thing to be noted is that out of 43, 11 industries have $d_3 \neq o$ or in other words d_3 is significantly different from zero at 5 per cent level of significance and for six more industries it is different from zero at 10 percent level of significance. That means 17 industries show constant returns to scale. Some of the important industries, which do not show constant returns to scale, are wood and wood products, paper and paper products, rubber and rubber products, drugs and pharmaceuticals, cement and its products, machine and tools, non-ferrous metals, railway rolling stocks, and generation, transmission and distribution of electricity. In this group of industries only one industry, machine and tools, shows Leontief type of production function. The rest of the industries of this group show either Cobb-Douglas or CES/VES trypes of production function. Industries based on agriculture and mining are showing the Cobb-Douglas form and the others CES/VES. The main industries which are not showing constant returns to scale but showing Cobb-Douglas form are silk and art silk manufactures, rubber products, and cement. Other industries showing Leontief type of production functions are edible oils (including hydrogenated oils) and chemicals and fertilizers which also show constant returns to scale.

Out of the 27 industries, which show CES/VES, 16 show constant returns to scale and 11 variable returns to scale. Two out of three Leontief type industries show constant returns to scale. Similarly, 8 out of 13 Cobb-Douglas type show constant returns to scale.

Elasticities estimated by the formula (4) show the following pattern. Fifteen industries have an elasticity coefficient of 1.0 or less than 1.0. Seventeen show an elasticity between 1.0 and 2.0 and rest of the eleven industries have an elasticity, 2.0 and above. The lowest elasticity turns to be .049 for machines and tools, and the highest for ships and boats (17.4631) which looks to be very high. The second highest elasticity is for sugar (5.4183).

Conclusions

It can be implied from the results about the elasticity of substitution that quite a large number of industries in India, have elasticities above unity which means there are ample opportunities of substituting labour for capital and thus sufficient employment may be generated in the economy so that the share of labour in the economy is not falling and hence the income distribution is not deteriorating. However, in the case of industries where elasticities are below unity, the scope for more employment generation is very limited.

These results also give support to the argument for developing the small-scale-sector even at the cost of providing subsidies and cheap loans (both in form of cash and raw materials and import licences), as these industries generally employ labour intensive technology which results in higher employment and more equitable income distribution. The same argument may be extended to bring into its fold the foreign trade sector; by arguing for the expansion of labour intensive exports and import-substitution (particularly in capital intensive industries).

REFERENCES

(1) Hicks, J.R.: The Theory of Wages , Macmillan, London, 1932.

(2) Allen, R.G.D.: Mathematical Analysis for Economists, Macmillan, 1938.

(3) Lu, Yao-chi and Fletcher, L.B.: "A Generalization of CES Production Function," The Review of Economics and Statistics, Vol. L., Nov. 1968.

(4) Arrow, K.J., Chenery, H.B., Minhas B.S. and Solow, R.M.: "Capital Labour Substitution and Economic Efficiency," The Review of Economics and Statistics, Vol. 43, No. 3, August, 1961.

(5) Lu, Y.C. and Fletcher, L.B., op.cit.

(6) Lu, Y.C.: "Variable Elasticity of Substitution Production Functions, Technical Change and Factor Shares," quoted in Lu and Fletcher, op.cit.

(7) Central Statistical Organisation: Annual Survey of Industries, 1966, Vol. 1 to 1e. Government of India, Calcutta.

(8) Yeung, P. and Tsang, H. "Generalised Production Function and Factor-Intensity Crossovers: An Empirical Analysis," The Economic Record, Vol. 48, No. 123, 1972.

(9) Yeung and Tsang, op.cit.

Table I: Production Function in Indian Manufacturing Industries - 1966

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
1 Flour Mills	.9261	.782 (2.1305)*							.228	.292
			.605 (1.54)**		.221 (1.155)			CES/VES	.250	.375
				.816 (2.471)*		.087 (.523)	-.292 (-2.48)*		.505	.629
2 Rice Mills	2.5950	2.295 (3.745)*							.650	.700
			2.140 (3.169)*		.097 (.790)			CES/VES	.619	.728
				2.324 (2.749)*		.141 (.790)	.073 (.646)	Constt. R.S.	.547	.741
3 Sugar	5.4185	1.608 (2.923)*							.387	.458
			1.728 (3.508)*		.397 (1.98)*			Cobb-Doglus	.515	.593
				1.695 (2.919)*		.411 (1.727)*	.014 (.128)	Constt. R.S.	.462	.597
4 Misc. Food Products	1.0240	1.047 (6.098)*							.858	.861
			.889 (2.181)*		.088 (.434)			CES/VES	.813	.866
				1.055 (2.015)*		.054 (.241)	-.115 (-.575)	Constt. R.S.	.784	.876

Contd..

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
5 Edible Oils	.4898	.488 (1.49)**							.087	.157
			.520 (1.266)		-.041 (-.159)			Leontief	.005	.158
				.282 (.618)		.165 (.480)	.155 (1.136)	Constt.	.031	.255
6 Tea & Coffee	2.5044	1.728 (4.020)*							.684	.729
			.936 (2.378)*		1.011 (2.928)*			Cobb-Douglas	.860	.900
				.936 (2.007)*		1.011 (1.906)*	.000045	Constt.	.826	.900
7 Tobacco & Prods.	.4464	.504 (1.321)							.085	.1996
			.143 (.886)		.384 (3.142)*			Cobb-Douglas	.597	.697
				.0989 (.557)		.403 (3.113)*	-.078	Constt. Rts.	.564	.728
8 Cotton Textiles	1.1436	.859 (7.637)*							.782	.795
			.919 (5.565)*		.0484 (.507)			VES/CES	.770	.799
				.912 (.408)		.049 (.4888)	.002 (.050)	Constt. Rts.	.753	.799

Contd..

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
9 Textile Dying, Bleaching etc.	2.2829	1.686 (7.659)*							.878	.893
			1.705 (7.024)*		.085 (.328)			VES/CES	.860	.895
				2.231 (6.150)*		.233 (.973)	-.180 (-1.777)	Constt. Rts.	.897	.936
10 Silk & Art Silk	2.3131	1.947 (11.803)*							.939	.946
			1.583 (9.756)*		.2075 (3.159)*			Cobb-Douglas	.971	.977
				1.502 (9.339)*		.224 (3.588)*	.0694 (1.448)**		.975	.983
11 Thread Ball, Carpet making etc.	.1190	.5897 (1.923)*							.268	.335
			.0584 (.232)		.301 (3.212)*			Cobb-Douglas	.621	.690
				.205 (.999)		.289 (3.928)*	.1487 (2.5802)*		.767	.831
12 Saw Milling and Plywood etc.	.8958	.9915 (2.866)*							.445	.507
			1.4446 (2.571)*		-.239 (-1.022)			VES/CES	.448	.571
				1.5337 (1.944)*		-.248 (-.964)	-.0354 (-.1762)	Constt. Rts.	.359	.373

Contd..

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
13. Wood & Furniture Manufacturing	.7892	.6983 (1.523)**							.141	.249
			.4778 (.6860)		.2752 (.4432)			VES/CES	.0302	.273
				1.45215 (1.846)*		-.4751 (-.716)	.5322 (1.859)*	Cobb-Douglas	.3119	.5699
14. Paper & Paper Products	1.4580	1.6188 (1.837)*							.192	.273
			1.7463 (1.748)*		-.1018 (-.345)			VES/CES	.104	.283
				.7106 (.779)		.0470 (.1921)	.3868 (2.351)*		.428	.5995
15. Printing & Photography etc.	3.4429	1.0515 (5.117)*							.611	.636
			.796 (3.6407)*		.2500 (2.171)*				.689	.727
				.696 (3.057)*		.216 (1.872)*	.0619 (1.279)	Cobb. Douglas Constt. Rts.	.702	.758
16. Leather & Leather	.6591	1.009 (3.116)*							.592	.560
			.9569 (3.289)*		-.2446 (-1.511)			Cobb- Douglas	.675	.784
				.9518 (3.134)*		-.2082 (-1.192)	.0866 (.8206)	Constt. Rts.	.647	.823

Contd..

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
17. Rubber Products	4.9314	1.3859 (3.641)*							.636	.688
			.4668 (1.239)		.607 (3.167)*				.855	.896
				-.177 (.-9246)		.7563 (9.403)*	.2232 (5.323)*	Cobb-Douglas	.978	.987
18. Chemicals & Fertilizers	.2111	.1114 (.228)							-.073	.004
			.3185 (.6366)		-.2554 (-1.3108)			Leontief's	-.016	.129
				.4806 (.7118)		-.246 (-1.208)	-.0814 (-.376)	Constt. Rts.	-.094	.140
19. Drugs & Pharmaceuticals	1.3681	1.5298 (5.614)*							.718	.741
			1.5087 (5.467)*		-.0725 (-.897)			VES/CES	.713	.761
				1.9818 (5.071)*		-.0494 (-.646)	-.16534 (-1.605)**		.751	.814
20. Refractories	.5085	.5250 (2.474)*							.363	.434
			.75296 (2.76)*		-.1267 (11.264)			VES/CES	.407	.539
				.4894 (1.908)*		.0006 (.0063)	.1884 (2.099)*		.601	.734

Contd..

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
21 Glas & Glass Ware	1.3113	1.3781 (13.366)*							.962	.968
			1.4884 (6.440)*		-.0451 (-.5421)			VES/CES	.957	.969
				1.5671 (7.196)*		-.0479 (-.631)	-.053 (-1.421)**		.964	.9797
22 Chinaware & Pottery	2.1232	1.6617 (3.234)*							.512	.567
			1.5611 (2.793)*		.0945 (.619)			VES/CES	.472	.589
				1.4235 (2.386)*		.0786 (.4381)	.1218 (.8203)	Constt. Rts.	.446	.631
23 Cement & its Products.	-.3413	.9063 (1.433)**							.087	.170
			-.02603 (-.116)		.6195 (9.458)*			Cobb.	.907	.924
				1.5671 (7.196)*		-.0479 (-.631)	-.053 (-1.421)		.964	.9797
24 Stone other minerals etc.	1.0992	1.09091 (7.9319)*							.8609	.875
			1.0477 (3.9102)*		.0237 (.1922)			Cobb. Douglas	.844	.875
				.8955 (2.2605)*		.1135 (.5140)	.0717 (.5441)	Constt. Rts.	.829	.880

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
25 Iron & Steel	.1849	.0736 (.440)							-.061	.0146
			.0968 (.6593)		.2365 (2.2335)*			Cobb. Douglas	.188	.304
				.1803 (1.19)		.3612 (2.4299)*	-.1214 (-1.177)	Constts. Rts.	.213	.382
26 Iron & Steel, Forging & Casting	.7114	1.3681 (1.4964)**							.087	.157
			2.3038 (1.902)*		-.5597 (-1.1566)			VES/CES	.112	.249
				2.1803 (1.6444)**		-.5596 (-1.0078)	.0723 XXXXX (.3095)	Constt. Rts.	.0325	.2558
27 Iron & Steel Structurals	1.3893	1.5864 (5.4869)*							.708	.732
			1.6328 (4.6555)*		-.0453 (-.2612)			VES/CES	.681	.734
				1.8489 (5.118)*		-.1493 (-.8385)	-.1525 (-1.4931)**		.718	.787
28 Non-Ferrous Basic Metals	2.1136	1.948 (5.4000)*							.8009	.829
			1.929 (4.986)*		.067 (.513)			VES/CES	.773	.838
				2.5561 (5.910)*		.054 (.524)	-.2385 (-2.0334)*		.8604	.920

Contd..

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
29 Metal Containers	.4738	.575 (2.141)*						VES/CES	.339	.433
			.929 (2.573)*		-.414 (-1.364)**				.422	.587
				.9903 (2.324)*		-.385 (-1.128)	-.0388 (-.391)	Constt. Rts.	.304	.602
30 Bolts & Nuts, etc.	1.4471	1.459 (3.795)*						VES/CES	.573	.615
			1.471 (2.353)*		-.007 (-.026)				.519	.615
				1.096 (1.606)**		.009 (.033)	.145 (1.212)	Const. Rts.	.546	.682
31 Metal Rous (Except Machinery	1.4072	1.709 (7.122)*						VES/CES	.793	.809
			1.909 (6.259)*		-.194 (-1.055)				.795	.826
				1.922 (5.677)*		-.196 (-1.012)	-.0086 (-.118)	Constt Rts.	.774	.826
32 Textile, Sugar and Tea Machinery	.4155	.3221 (1.537)**						VES/CES	.146	.252
			.241 (1.083)		.1736 (1.029)				.153	.365
				.232 (.959)		.1573 (.846)	.027 (.417)	Const. Rts.	.0135	.386

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
33 Machine & Tools	.0429	-.227 (-.266)							-.102	.008
			.0717 (.069)		-.193 (-.541)			Leontief	-.197	.043
				-.8493 (-.833)		-.342 (-1.081)	.530 (1.938)*		.110	.377
34 Agricultural Machinery	1.1588	1.705 (5.673)*						VES/CES	.776	.801
			1.598 (4.899)*		-.185 (-.903)				.771	.822
				1.597 (4.589)*		-.227 (-.931)	-.058 (-.388)	Constt. Rts.	.739	.826
35 Steam Engines & Turbains	1.1075	.9445 (3.421)*						VES/CES	.543	.594
			.947 (3.259)*		.046 (.479)				.494	.607
				.8738 (2.215)*		.033 (.297)	.035 (.301)	Constt. Rts.	.419	.613
36 Elect. Cables, Wires,	1.3605	.959 (1.917)*							.229	.315
			.6358 (1.300)		.346 (1.692)**			Cobb. Douglas	.375	.514
				1.411 (2.047)*		.2414 (1.199)	-.233 (-1.488)**		.467	.645

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Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
37 Batteries & Radios	1.0554	.974 (5.107)*						CES/VES	.781	.813
			.8524 (1.9897)*		.1024 (.3247)				.744	.817
				.848 (1.758)*		.0937 (.2486)	.0136 (.064)	Constt. Rts.	.6798	.817
38 Ships & Boats	17.4631	.8328 (3.754)*						CES/VES	.621	.668
			.7868 (3.659)*		.1605 (1.269)				.651	.738
				.7776 (2.919)*		.1565 (1.077)	.007 (.907)	Constt. Rts.	.582	.739
39 Railways & Rolling Stock	1.0897	1.2116 (6.3135)*							.795	.816
			1.2192 (5.976)*		-.0168 (-.298)			CES/VES	.772	.818
				.8965 (4.287)*		-.0233 (-.5227)	.0712 (2.418)*		.858	.9007
40 Man. of Motor Vehicles	1.3714	1.3568 (4.219)*							.604	.640
			1.2192 (5.976)*		-.0168 (-.298)			CES/VES	.772	.818
				1.8265 (2.712)*		.1092 (.412)	-.2195 (-1.0674)	Constt. Rts.	.567	.685

Industry	Elasticity of substitution	b_1	b_2	b_3	c_2	c_3	d_3	Form of the function	$\bar{R}^2$	R^2
41 Repair of Motor Vehicles	1.1220	1.2714 (8.911)*							.813	.824
			1.2859 (8.683)*		-.0189 (-.547)			CES/VES	.805	.827
				1.2214 (8.719)*		-.0141 (-.422)	.0344 (1.5193)**		.820	.850
42 Motor Cycles and Bicycles	1.8114	.8585 (3.84)*						Cobb. Douglas	.632	.678
			.6414 (3.352)*		.299 (2.473)*				.787	.840
				.8001 (2.125)*		.274 (1.973)*	-.064 (-.502)	Constt. Rts.	.757	.848
43 Gen. Trans. & Dist. of Electricity	2.0318	1.6096 (2.057)*						CES/VES	.177	.232
			1.5141 (1.803)		.1625 (.410)				.125	.242
				1.5679 (2.491)*		-.241 (-.75)	.5196 (3.341)*		.509	.607

Figures in brackets are t-values.

* significant at 5% level of significance

** significant at 10% level of significance.