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THE ISOLATION PARADOX AND THE DISCOUNT RATE FOR BENEFIT-COST ANALYSIS

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I. INTRODUCTION

A dominant issue in the literature on benefit-cost analysis has been the appropriateness or otherwise of using market rates of discount for the intertemporal aggregation of benefits and costs. One important argument for using a non-market rate of discount is based on the insight that under certain assumptions individuals would voluntarily enter into a social contract committing them to increase their total savings, for the benefit of future generations, above the level they chose privately. This divergence of collective and individual behavior, which is a manifestation of the famous "prisoner's dilemma" problem of game theory, was postulated by Baumol [1952] and Eckstein [1958] and was called the "isolation paradox" by Sen [1961] in a study of optimal saving.

In two celebrated papers Marglin [1963a, 1963b] argued that in an economy containing both private saving for benevolent purposes and public investment the existence of an isolation paradox can justify the use of a discount rate for benefit-cost analysis which is below the private rate of return on savings. This argument is now widely recognized in the literature on benefit-cost methodology[1], with major criticisms concentrating on the empirical validity of the assumptions about individual preferences postulated in the formal development of the model in Sen [1967].[2] Since the parameters concerned are difficult to measure, the likelihood that the isolation paradox will indeed exist has become largely a matter of individual judgement. On the other hand, there has been essentially no challenge to the conclusion that if an isolation paradox of the Marglin-Sen type does exist the appropriate rate of discount for use in benefit-cost analysis lies below the market rate of discount.

The present paper takes issue with this conclusion. Taking the isolation paradox argument on its own terms, it aims to show that the appropriate rate of discount for use in benefit-cost analysis is the market rate of discount, whether the isolation paradox actually holds or not. This is done by constructing a simple general equilibrium model reflecting the inter-generational benevolence lying at the heart of the isolation paradox analysis. Like Marglin and Sen, we ignore other capital market distortions. This model is presented in Section II. In Section III it is then shown that the appropriate rate of discount for use in benefit-cost analysis in this model is the market rate of discount, regardless of whether the Marglin-Sen assumptions on individual preferences and distributional mechanisms, implying the existence of an isolation paradox, are imposed on the model. The alternative approach advocated by Marglin [1936b] and Dasgupta, Marglin and Sen [1972] imposes far more formidable data requirements and could at best produce results which are equivalent to the calculation of net present value using the market rate of discount; but in Section IV we argue that in general no such equivalence is possible.

II. PRIVATE SAVINGS AND THE ISOLATION PARADOX

Structure of the Model

We begin with a simple general equilibrium model which captures the essence of the problem. The well known problems of generalizing from two period savings models[3] require us to begin with three periods (generations), denoted 0, 1 and 2. Since the focus of the paper is on inter-generational rather than intragenerational savings, we imagine the life of each generation to be focused on a single discrete point in time and, for simplicity, the interval between generations will correspond to a fixed period of time. Intra-generational savings are thereby ignored. The existence of an isolation paradox in any generation requires that that generation contain a minimum of two individuals. So the first two generations, 0 and 1, each contain two individuals, denoted 1 and 2 and 3 and 4, respectively. But since the isolation paradox cannot arise for the final generation (there is no subsequent generation for whom to save), it will contain only one individual, denoted 5.

Each individual receives an income in lump sum form. He can then in principle make donations of two types: (i) to his contemporary, or (ii) to members of the next generation. The individual in the final period is an exception since, for him, donations of both types are infeasible. Type (ii) donations earn a rate of return ρ and the proceeds are divided among the members of the succeeding generation. We assume that this rate of retur ρ is unaffected by any of the marginal savings decisions or public investment projects occurring within the model and for simplicity we will suppose it to be constant over time. It is convenient to think of ρ of as the rate of return available on an external capital market. The way type (ii) donations are divided among members of the next generation is, for now, left open. It may be determined by the donor himself (say, through a will), by some other, more rigid rule of distribution beyond his control, or by some combination of the two, but the donor is nevertheless aware of the way his donations are to be distributed. In keeping with the Marglin-Sen assumptions, it will presently be assumed that individual preferences are such that type (i) donations never occur while (except for period 2) all individuals make positive type (ii) donations.

The utility of each individual depends on his own consumption, that of his contemporary and that of the members of the succeeding generation. The individual in the final generation is again the obvious exception. So for individual 1, for example, $U^1 = U^1(c^1, c^2, c^3, c^4)$, where c^i denotes the consumption of individual i. The utility of the individual in period 2 depends simply on his own consumption. The utility function of each individual is strictly concave, twice differentiable and strictly increasing in all arguments with the possible exception of the consumption of his contemporary, in which it is non-decreasing. Hence, there is no malevolence.

The consumption of individual 1 is given by $c^1 = Y^1 - h^1 + h^2 - s^1$, where Y^1 is his lump sum income, h^1 is his voluntary donation to his contemporary, individual 2, h^2 is his contemporary's donation to him, and $s^1 = s^{13} + s^{14}$ is his total voluntary saving for the benefit of the next generation, s^{13} and s^{14} being those portions of it earmarked for individuals 3 and 4, respectively. The consumption of individual 3 is $c^3 = Y^3 - h^3 + h^4 - s^3 + (1 + \rho)(s^{13} + s^{23})$, where all terms are defined analogously as above. The final term indicates the savings contributions of individuals 1 and 2 to individual 3 (s^{13} and s^{23}, respectively) which then earn the rate of return ρ. For individual 5, $c^5 = Y^5 + (1 + \rho)(s^3 + s^4)$. The expressions for the consumption of individuals 2 and 4 are directly analogous with those above for 1 and 3, respectively.

Private Savings

Consider now the private utility maximization problem of individual 1, taking the behavior of all others individuals (in particular, his contemporary) as given. The Kuhn-Tucker conditions are

$$h^1 (U_1^1 - U_2^1) = 0; \; U_1^1 - U_2^1 \geqq 0 \tag{1}$$

and

$$s^1 (U_1^1 - (1 + \rho) U_s^1) = 0; \; U_1^1 - (1 + \rho) U_s^1 \geqq 0, \tag{2}$$

where $U_j^1 \equiv \partial U^1/\partial c^j$ and U_s^1 is a shorthand notation for $(\lambda^{13}U_3^1 + (1 - \lambda^{13}) U_4^1)$. Obviously, λ^{13} denotes the marginal proportion of individual 1's savings earmarked for individual 3, which may be either a choice variable or a parameter for individual 1.

In parallel with the Marglin-Sen assumptions, we assume that individual preferences are such that these conditions are satisfied by $h^1 = 0$ and $s^1 > 0$, implying that $U_1^1 > U_2^1$

and

$$U_1^1 = (1 + \rho) (\lambda^{13}U_3^1 + (1 - \lambda^{13}) U_4^1). \tag{3}$$

The story for individuals 2, 3 and 4 is identical. No one contributes voluntarily to his contemporary, but each saves voluntarily for the benefit of his successors. These future benefits are discounted by each individual at the rate ρ, which is called alternatively the private rate of discount (return) or market rate of discount (return).

The Isolation Paradox

Imagine individuals 1 and 2 to have separately chosen their optimal levels of savings, behaving atomistically as above. We now consider a contract between them which commits each to raise his total level of savings by one unit. These additional savings earn the rate of return ρ as before and the proceeds are distributed to individuals 3 and 4 in the proportions γ^3 and $1 - \gamma^3$, respectively. The effect on individual 1's welfare is given by

$$dU^1 = - U_1^1 - U_2^1 + 2 (1 + \rho) (\gamma^3 U_3^1 + (1 - \gamma^3) U_4^1) . \qquad (4)$$

Similarly,

$$dU^2 = - U_2^2 - U_1^2 + 2(1 + \rho)(\gamma^3 U_3^2 + (1 - \gamma^3) U_4^2) . \qquad (5)$$

The debate in the literature has centered on whether or not it is reasonable to expect both the private optimal savings conditions derived above and $dU^1 > 0$ and $dU^2 > 0$ in (4) and (5) to hold. If they do, the isolation paradox holds, meaning that the initial equilibrium under private savings was not Pareto optimal in what Marglin calls the "bourgeois democratic" sense namely that the welfare of only the current generation (generation 0) is considered.[4] Since strategic behavior is ruled out, adoption of the above contract would be supported by a consensus of generation 0.

In a masterly paper, Sen (1967) sets out several sets of sufficient but not necessary conditions for this to be so. By substituting the optimal private savings condition (3) into (4), we see that $dU^1 > 0$ is equivalent to

$$U_1^1 - U_2^1 > 2(1 + \rho) \quad (\lambda^{13} - \gamma^3) U_3^1 + (\gamma^3 - \lambda^{13}) U_4^1 . \qquad (6)$$

Among Sen's sufficient conditions are the obvious ones of $\lambda^{13} = \gamma^3$ and $U_3^1 = U_4^1$, either of which guarantees (6) since, also by assumption,[5] $U_1^1 > U_2^1$. (Analogous conditions ensure that $dU^2 > 0$.) Neither of these special cases is as far-fetched, as it first appears. This is best seen by reconsidering the meaning of λ^{13}. If λ^{13} is a choice variable for individual 1, then provided both s^{13} and $s^{14} > 0$, he chooses λ^{13} such that $U_3^1 = U_4^1$. So the magnitude of γ^3 makes no difference to him for marginal collective decisions and (6) holds immediately. On the other hand, if λ^{13} is a rigidly specified rule of distribution, outside the control of individual 1, it is possible that $U_3^1 \neq U_4^1$. But if the same rigid rule applies to the distribution of collective savings as to private savings, as seems possible at least, $\lambda^{13} = \gamma^3$ and again (6) follows.

As several subsequent authors have pointed out, relation (6), and the corresponding relations for individuals 2, 3 and 4, may or may not in fact hold, and this matter is not easily resolved empirically. We do not propose to join the debate on this issue since it is our aim in the next section to show that whether these inequalities (or the opposite ones) hold or not makes no difference for the choice of the appropriate rate of discount for benefit-cost analysis. But for the moment, suppose that these inequalities do hold, as in (6). Consider a collective contract of the above type earning a rate of return r^1, rather than ρ, such that $dU^1 = 0$. Solving for r^1 we obtain

$$r^1 = \tfrac{1}{2}(U_1^1 + U_2^1)[\gamma^3 U_3^1 + (1 - \gamma^3)U_4^1]^{-1} - 1, \qquad (7)$$

which, with either $\gamma^3 = \lambda^{13}$ or $U_3^1 = U_4^1$ becomes, utilizing (3),

$$r^1 = \frac{1}{2} (1 + U_2^1/U_1^1) (1 + \rho) - 1 . \tag{8}$$

This is what Marglin and Sen each call the "social" rate of discount. This discount rate is not reflected in individual market behavior, as distinct from ρ, which is called the "private" rate of discount. Given the Marglin-Sen assumption that $U_1^1 > U_2^1$, from (6) and (8) we have $r^1 < \rho$ and likewise, performing this exercize for individuals 2, 3 and 4, we have $r^i < \rho$, $i = 1, \ldots, 4$; but there is nothing to ensure $r^i = r^j$, $i \neq j$. There is a value of r^i for each individual in each generation and strong additional assumptions are needed to guarantee that they are the same. It seems somewhat odd to call such an individual discount rate the "social" rate when it is, in a very real sense, more "private" than ρ, the market rate. Nevertheless, r^i applies to collective savings decisions, while ρ applies to individual ones. Furthermore, however the aggregation problem of moving from the set $\{r^i\}$ to "the" social rate of discount, r, is resolved, it seems clear that since $r^i < \rho$ for all i, $r < \rho$ as well. Acting collectively, it seems, the members of society are prepared to undertake investments that, acting individually, they are not. [6]

To see the significance of r^i, imagine the introduction of a small public project which, to keep the example simple, affects the consumption of the two individuals in each generation equally. So $dc^1 = dc^2 = dC_o/2$, $dc^3 = dc^4 = dC_1/2$, and $dc^5 = dC_2$, where dC_t denotes the change in total consumption in generation t. Consider its effects on the utility of individual 1.

$$dU^1 = \sum_{i=1}^{4} U_i^1 dc^i = (U_1^1 + U_2^1)\, dC_o/2 + (U_3^1 + U_4^1)\, dC_1/2 , \tag{9}$$

$$\frac{dU^1}{U_1^1} = \frac{(1 + r^1)}{(1 + \rho)} dC_o + \frac{1}{(1 + \rho)} dC_1 . \tag{10}$$

Suppose we have somehow determined the changes in final consumption in each period induced by the project. To determine whether individual 1 has been made better off or worse off, it seems from (10) that the weight to be applied to the change in consumption in period 1 relative to that in period 0 is $1/(1 + r^1)$. The appropriate rate of discount, from individual 1's point of view, is r^1. Suppose,[7] following Dasgupta, Marglin and Sen [1972] that $r^1 = r^2 \equiv r$. Then it seems that if we grant the existence of the isolation paradox, implying $r < \rho$, there is good reason for thinking that the appropriate discount rate for benefit-cost analysis is the "social" rate of discount, r, a conclusion that has been widely accepted in the literature. But we shall now show that this argument is erroneous.

III. GENERAL EQUILIBRIUM EFFECTS OF A PROJECT

Suppose now that a "small" public (or private) investment project is adopted and can be described by a vector of net returns to the five individuals, spread out over the three periods.[8] If all these net returns were positive (negative) the project would be unambiguously desirable (undersirable) and benefit-cost analysis would be unnecessary. To keep the problem non-trivial we suppose that at least one of these net returns is negative and at least one is positive. Let the net returns in period 0 from project x sum to B_0^x, divided among individuals 1 and 2 in the proportions α_0^x and $1 - \alpha_0^x$, respectively. Similarly, in period 1 these net returns sum to B_1^x, divided between individuals 3 and 4 in the proportions α_1^x and $1 - \alpha_1^x$ and in period 2 the net return is B_2^x. There are no sign restrictions on the total returns to any one generation or on the proportions in which they are divided. For example, B_o^x may be negative (probably the typical case) but α_o^x also negative, so that individual 1's net return is positive while individual 2's is negative. We now wish to consider the adjustments that follow this.

We wish to derive the change in the final consumption of each individual resulting from the adoption of the project. For the five individuals this gives:

$$dc^1 = - ds^1 + \alpha_0^x B_0^x, \tag{11}$$

$$dc^2 = - ds^2 + (1 - \alpha_0^x) B_0^x, \tag{12}$$

$$dc^3 = (1 + \rho) (\lambda^{13} ds^1 + \lambda^{23} ds^2) - ds^3 + \alpha_1^x B_1^x, \tag{13}$$

$$dc^4 = (1 + \rho) ((1 - \lambda^{13}) ds^1 + (1 - \lambda^{23}) ds^2) - ds^4 + (1 - \alpha_1^x) B_1^x, \tag{14}$$

and

$$dc^5 = (1 + \rho)(ds^3 + ds^4) + B_2^x \, . \tag{15}$$

Now divide (13) and (14) by $(1 + \rho)$ and divide (15) by $(1 + \rho)^2$ and sum the six equations. This gives

$$dc^1 + dc^2 + \frac{(dc^3 + dc^4)}{1 + \rho} + \frac{dc^5}{(1 + \rho)^2} = B_0^x + \frac{B_1^x}{(1 + \rho)} + \frac{B_2^x}{(1 + \rho)^2} \, . \tag{16}$$

Thus one constraint that the adjustment of consumption levels must satisfy is that the net present value of the stream of consumption changes, discounted at the rate ρ, must be equal to the net present value of the returns of the project, also discounted at the rate ρ. We shall refer to the latter, the right hand side of (16), as N_ρ^x. Next, there are four equilibrium conditions relating to the voluntary donations of individuals 1 throught 4, described for individual 1 by (3), which must also be satisfied if these individuals are to have positive savings before and after the project is adopted. Differentiating these equations totally and incorporating (16) we obtain the system:

$$\begin{bmatrix} 1 & 1 & 1/(1+\rho) & 1/(1+\rho) & 1/(1+\rho)^2 \\ J_1^1 & J_2^1 & J_3^1 & J_4^1 & 0 \\ J_1^2 & J_2^2 & J_3^2 & J_4^2 & 0 \\ 0 & 0 & J_3^3 & J_4^3 & J_5^3 \\ 0 & 0 & J_3^4 & J_4^4 & J_5^4 \end{bmatrix} \begin{bmatrix} dc^1 \\ dc^2 \\ dc^3 \\ dc^4 \\ dc^5 \end{bmatrix} = \begin{bmatrix} N_\rho^x \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{17}$$

For $i = 1, 2$, $J_k^i \equiv U_{ik}^i - (1 + \rho)(\lambda^{i3} U_{3k}^i + (1 - \lambda^{i3}) U_{4k}^i)$, $k = 1, \ldots, 4$ and for $i = 3, 4$, $J_k^i \equiv U_{ik}^i - (1 + \rho) U_{i5}^i$, $k = 3, 4, 5$.

The solutions to this system are given by

$$dc^i = N_\rho^x R^i, \ i = 1, \ldots, 5. \tag{18}$$

Writing J for the square matrix in (17), $R^i = J^{1i}/|J|$, where J^{1i} is the cofactor of the element in the first row and ith column of J and $|J|$ is the determinant of J. The general equilibrium change in the consumption of individual i due to the project is a constant, R^i, multiplied by the net present value of the project discounted at the rate ρ, N_ρ^x. Note in particular that the R^i terms contain <u>no</u> project-specific data. The characteristics of the project enter the determination of the dc^i's only <u>via</u> the N_ρ^x terms. With some relatively weak restrictions on individual utility functions it is possible to ensure that each of the R^i terms is strictly positive. It must be stressed that these restrictions are sufficient but not necessary for the strict positivity of the R^i terms and that none is in any way inconsistent with the existence of an isolation paradox.

First, suppose the utility functions are additively separable. This together with our assumption of strict concavity, implies $U^i_{jk} = 0$, $j \neq k$ and $U^i_{kk} < 0$ (assuming c^k is an argument of u^i, of course). We then have $J^i_i < 0$ and $J^i_k > 0$ except that $J^i_k = 0$ when i and k are contempararies. This guarantees $R^i > 0$ for all i, and is perfectly consistent with the existence (or non-existence) of the isolation paradox, but it is considerably stronger than we require. Next, suppose (i) that utility functions are additively separable between the consumption of that individual's contemporary and the other arguments of the function and (ii) that for i = 1, 2 $(\lambda^{i3} U^i_{3K} + (1 - \lambda^{i3}) U^i_{4K}) \leq 0$, k = 3, 4. Assumption (i) means that the utility function of individual 1, for example, can be written $U^1(c^1, c^2, c^3, c^4) = f^1(c^1, c^3, c^4) + g^1(c^2)$. This is of course, consistent with the isolation paradox (imagine the extreme case where $dg^1(c^2)/dc^2 = 0$, representing indifference towards one's contemporary) and ensures again that $J^i_k = 0$, where i and k are contemporaries. Assumption (ii) is again consistent with the isolation paradox and with our concavity assumptions and implies that $J^i_k > 0$, where k belongs to the generation following i. Together, these assumptions imply $R^i > 0$ for all i.

Any set of assumptions implying $J^i_i < 0$, $J^i_k \geq 0$, and $|J^i_i| > |J^i_k|$ where i and k are contemporaries and $J^i_k > 0$, where k belongs to the generation following i, is sufficient to ensure the positivity of the R^i terms. These assumptions seem "reasonable", and there is no inconsistency between them and our concavity assumptions or with the isolation paradox. So grant for the moment that each of the R^i terms if positive. This means that, from (18), the sign of the change in the consumption of each individual[9] is the same as the sign of N^x_ρ. Furthermore, the sign of the change in the utility of each individual is the same as N^x_ρ. For individual 1, for example,

$$dU^1 = \sum_{k=1}^{4} U^1_k \, dc^k = N^x_\rho \sum_{k=1}^{4} U^1_k \, R^k \equiv N^x_\rho M^1 . \qquad (19)$$

Since $U^1_k \geq 0$ for all k (no malevolence), $M^1 > 0$ and individual 1 benefits or loses from the project as N^x_ρ is positive or negative. The same applies to the other four individuals,

$$dU^i = N^x_\rho M^i, \qquad i = 1, \ldots, 5 \qquad (20)$$

where $M^i > 0$ and, as with R^i, M^i contains no project-specific data.

Now consider what is changed if the assumptions required in (6) for the existence of the isolation paradox are granted. Suppose the isolation paradox holds for all individuals in the first two generations. Still, the sign of the change in utility of each person due to a project is the same as that of every other, namely the sign of N^x_ρ. Whether the isolation paradox holds or not, discounting the aggregate returns from the project in each year at the market rate of discount provides an adequate indication of the welfare effects of the project.[10] Clearly, if we have several mutually exclusive projects, provided $R^i > 0$ larger values of N^x_ρ imply larger values of dc^i and dU^i; the projects can be ranked according to N^x_ρ. Furthermore, incorporation of income distributional considerations into the choice of projects, to the extent that it leads to the selection of projects with lower values of N_ρ^x, is clearly undesirable from the point of view of each individual, regardless of his initial lump-sum income.

Now return to the issue of the signs of the R^i terms. Restrictions on individual preferences which, in our view, are "weak", guarantee $R^i > 0$, but examples not satisfying them and implying the negativity of one or more of the R^i terms could presumably be constructed. What would this mean? Consider the change in utility of individual 1 resulting from a project, as given by (19). The negativity of one or more of the R^i terms does not necessarily imply the negativity of M^1, or similarly of M^2. It is easilv shown that $M^i < 0$ implies that individual i is made worse off by, for example, an increase in the lump sum income of each individual (Y^k, where $k = 1, \ldots, 5$), including himself. He is similarly harmed by an increase in his own income alone. This possibility seems sufficiently pathological to be disregarded, but it is clearly impossible for all the M^i terms to be negative. This would be inconsistent with the absence of malevolence in individual utility functions.

It should be clear that the possibility that a project with $N_\rho^x > 0$ could generate $dU^i < 0$ for some i rests on the nature of individual preferences and not on the distributional impact of the project. Suppose, though, that M^1 and M^2 have opposite signs and that projects are to be evaluated from the point of view of the present (initial) generation. Consider a Bergson-Samuelson social welfare function $W = W(U^1, U^2)$, with the derivatives $W_1, W_2 > 0$. Then

$$dW = \sum_{i=1}^{2} W_i dU^i = N_\rho^x \sum_{i=1}^{2} W_i M^i \equiv N_\rho^x V. \tag{21}$$

Provided $V > 0$, projects can be ranked simply by examing N_ρ^x. Knowledge of the precise form of W is not required. In the bizarre case $V < 0$ this would still be true, except that the rankings would be inverted, a possibility that hardly seems interesting.[11] We conclude that when there is inter-generational benevolence projects can be ranked according to their net present value at the market rate of discount, whether the isolation paradox holds or not.

Clearly, the very weak restrictions ensuring that M^1 and M^2 have the same sign do not imply that $r^1 = r^2$. If we assume either that $\lambda^{13} = \lambda^{23} = \gamma^3$, or that λ^{13} and λ^{23} are control variables for individuals 1 and 2, either of which implies the existence of an isolation paradox for both individuals 1 and 2, $r^1 = r^2$ requires that $U_2^1/U_1^1 = U_1^2/U_2^2$. This is not implied by, in fact has very little to do with, the weak requirements for M^1 and M^2 to have the same sign. Even though M^1 and M^2 have the same sign, r^1 and r^2 will still typically differ. But if sign (M^1) = sign (M^2) any project that benefits individual 1 also benefits individual 2, and vice versa. Despite the fact that $r^1 \neq r^2$ it is not possible to construct a project that harms one and benefits the other, the implications of the isolation paradox argument notwithstanding.

What underlies these results is a "smoothing" of the impact of public projects both within and between generations via the private donations of individuals. If one generation or individual is initially affected adversely by a project, even though $N_\rho^x > 0$, this is compensated for by a contraction in the voluntary donations of those individuals to the next generation, so as to restore the donor's private savings equilibrium (given, for individual 1, by (3)). Individuals affected disproportionately favorably respond by increasing their voluntary donations until their private savings equilibrium is restored. The first constraint on these adjustments is given by (16); the net present value of the stream of changes in consumption must sum to the net present value of the project, both discounted at the market rate of discount.[12] These two aspects of the problem, the "smoothing out" effect implied by the adjustments restoring a private savings equilibrium and the constraint on this process given by the net present value of the project, have not been explicitly incorporated into the analysis in the earlier literature, which was essentially partial equilibrium in character. The failure to view the impact of public projects within a general equilibrium context has led to a critical error.

A second set of constraints on the above adjustments is given by the assumption that individuals 1 through 4 have positive consumption and savings, both before and after the project is introduced. That is

$$s^i + ds^i > 0, \qquad i = 1, \ldots, 4. \tag{22}$$

and

$$c^i + dy^i - ds^i > 0, \qquad i = 1, \ldots, 5. \tag{23}$$

where dy^i is a shorthand notation for the change in individual i's income due to the project and due to changes in his receipts for the previous generation. We assume that the project is sufficiently "small" that positivity constraints (22) and (23) are not violated. The adjustments of private voluntary savings required to restore the private savings equilibrium, as induced by the project, are all assumed to be feasible. If the project was "large" and the difference between the initial impact of the project on each individual and its final general equilibrium impact was also large, some of the required adjustments could be infeasible.

Finally, we consider the generalization of our results. Our results extend immediately to n generations. So far as the first n-1 generations are concerned, no restrictions on the number of individuals involved in each, or on the way the total net returns to generation t, B_t, is distributed within that generation, need be introduced. This is seen readily be examining equations (11) to (16). The first n-1 generations may contain $m_1, m_2, \ldots, m_{n-1}$ individuals, and the net returns to each generation and the savings from the previous generations may each be distributed in any way (not necessarily related). Summing the m_t equations defining the change in consumption of the members of generation t eliminates all distributional parameters as before. The resulting equations may then each be divided by $(1 + \rho)^t$ and summed as before; the conclusions are unaltered.

A difficulty arises when the final generation is allowed to contain more than one individual and the proportions in which the savings of the previous generation are distributed among these individuals differ from the proportions in which the total project returns to the final generation, B_n, are distributed. For a sufficiently large divergence between the two, there may be no way that the private donations of the previous generation can "smooth out" the equilibrium consumption changes of the members of the final generation sufficiently to guarantee that they all have the same sign. This problem arises only for the final generation and occurs because that generation necessarily has no savings variable itself which it can adjust, a problem which is essentially an artifact of using finite period models. Nevertheless, to guarantee that all the dc^i variables for the members of the final generation have the same sign it is necessary to introduce a separate redistributional mechanism (for the final generation alone) or to impose the restriction that the savings of the previous generation and the returns from the project are distributed among the members of the final generation in the same non-negative proportions.

IV. THE "SOCIAL" RATE OF DISCOUNT AND THE SHADOW PRICE OF CAPITAL

The existence of an isolation paradox of the type identified by Eckstein, Sen and Marglin has been used as a rationale for a benefit-cost methodology differing in two essential ways from the calculation of N_ρ^x as above. The first difference concerns the rate of discount, as we have explained in Section II. The second concerns the introduction of a "shadow price of capital". This parameter reflects the value of consumption, present and future, foregone by drawing the necessary capital into the public sector to set up the project. In Marglin [1963b] and Dasgupta, Marglin and Sen [1972] the calculation of the shadow price of capital is discussed at length. All the expressions presented are described as approximations to the appropriate shadow price, even though most are quite complicated, and we do not propose to discuss their details here.[13] All depend, in various ways, on the private rate of discount, ρ, and the "social" rate of discount, r, and have the property that for $r < \rho$, $S^K \geq 1$. What is clear is that the shadow price of capital is the same for all "small" projects, assuming they are financed in the same way.

In the previous section we argued that when there is inter-generational benvolence, and whether the isolation paradox holds or not, projects can be ranked according to their welfare effects by means of their net present value at the market rate of discount. At best, any alternative benefit-cost analysis procedure will give equivalent results. The question we wish to raise is whether the "social" rate of discount/shadow price of capital methodology outlined above can in principle give results equivalent to those obtained with N_ρ^x. Denote the shadow price of capital S^K. Then the two procedures we are considering amount to

(assuming that $B_o < 0$ and $B_1, B_2 \geqq 0$)

$$N_\rho^x = \sum_{t=o}^{2} B_t/(1+\rho)^t \tag{24}$$

and

$$V^x_{r,S^K} = S^K B_o + \sum_{t=1}^{2} B_t/(1+r)^t \cdot \tag{25}$$

Table 1

Details of Hypothetical Projects

Project	B_o	B_1	B_2
a	-1	0	4
b	-1	2	0

Suppose, for example, that $\rho = 1$. Then referring to projects a and b in Table 1 (assumed "small"), we find using (24) that $N_\rho^a = N_\rho^b = 0$. As we have shown, it follows that individuals 1 and 2 are indifferent between accepting or rejecting either project. To give equivalent results, application of (25) must give

$$-S^K + 4(1+r)^{-2} = 0 \tag{25a}$$

and

$$-S^K + 2(1+r)^{-1} = 0 \cdot \tag{25b}$$

For $r > -1$ these equations have the unique solution $r = 1$ and $S^K = 1$; if $-1 < r < 1$ in both (25a) and (25b), then S^K cannot have the same value for the two projects. For example, $r = 1/2$ implies values of S^K of 16/9 and 8 in (25a) and (25b), respectively. Expression (25) cannot yield the correct selection criterion when r differs from ρ unless either the shadow price of capital or the "social" rate of discount is "tailor-made" for each of the two projects. Clearly (21) fails as a decentralized evaluation methodology. It cannot properly capture the general equilibrium welfare impact of a small project in an economy where the isolation paradox holds.

IV. CONCLUSIONS

This paper has employed a simple general equilibrium model of inter-generational benevolence to examine the validity of a widely accepted claim based on the new-famous "prisoner's dilemma" problem. This claim is that the existence of a special form of inter-generational benevolence, known as the isolation paradox case, implies that the rate of discount used in benefit-cost analysis should be below the market (private) rate of discount. We have found that the argument cannot be sustained and have attempted to show that the appropriate rate of discount is the market rate, whether the isolation paradox exists or not. Under relatively weak assumptions, the existence of inter-generational benevolence implies that the calculation of the net present value of a project at the market rate of discount provides an unambiguous indicator of the effects of the project on the welfare of <u>each</u> individual, regardless of the distributional impact of the project, a much stronger result than can be shown in the absence of inter-generational benevolence. The case for this benefit-cost rule is strengthened, rather than weakened, by the existence of inter-generational transfers, even when the isolation paradox holds. Alternative benefit-cost procedures, involving the calculation of a "social rate of discount" and a "shadow price of capital" are informationally more costly and could at best provide equivalent results; but we have attempted to show that such an equivalence is not possible in general.

Finally, we wish to make it clear that the results of this paper cast no doubt on the analytical validity, intellectual interest, or potential social importance of the isolation paradox argument <u>itself</u>, or of other, similar forms of the "prisoner's dilemma" framework. What we have questioned is one particular, but important, application of this analysis: the claim that, in an economy where private savings for benevolent purposes and public (or private) investment coexist, projects should be discounted at a rate below the market rate of discount. It remains true that if the isolation paradox holds, the equilibrium under private savings is not Pareto optimal; but discounting public (or private) investment at a rate other than the market rate of discount does not represent an opportunity for achieving a welfare gain. Public policies seldom, if ever, resemble the form of all-embracing social contract envisaged in the isolation paradox argument (specifying that each person increase his <u>total</u> savings by one unit). Public policies are superimposed on the actions of private decision-makers, who then adjust, and when these adjustments are taken into account the story changes dramatically.

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FOOTNOTES

[1]For example, see Prest and Turvey [1965] pp. 697-698, Dasgupta, Marglin and Sen [1972], p. 160, Layard [1972] pp. 37-39, Herfindahl and Kneese [1974], pp. 206-209, Mishan [1976] pp. 206-207, Boadway [1978] pp. 265-266, Boadway [1979] pp. 198-200.

[2]See Tullock [1964], Lind [1964], Usher [1964], Lecomber [1977] and Wellisz [1977].

[3]As Dasgupta, Marglin and Sen [1972] point out (pp. 162, 174), many investment rules that are equivalent in two period models are not equivalent in models with more than two periods. The generalization of our results is discussed at the end of Section III.

[4]The analogy between savings as treated here and a public good in the Samuelsonian sense is discussed in Sen [1972] and its externality character is discussed in Dasgupta, Marglin and Sen [1972], p. 160.

[5]It should be clear that it is not simply the <u>existence</u> of inter-generational benevolence that is the source of the isolation paradox but its presence combined with the relative <u>absence</u> of intra-generational benevolence.

[6]Eckstein [1958] pp. 99-100 presented the essence of this argument, which later became known as the "isolation paradox," in support of a discount rate for evaluating natural resource projects below the market rate of interest: "It is not logically inconsistent for the same person to be willing to borrow at high interest rates to increase his present consumption while voting to spend tax money to build a project from which future generations will benefit, for in the case of a vote to tax, he can be sure that the other individuals in the society will be compelled to act similarly"

[7]Recalling that r^1 and r^2 can differ, it also appears from (10) that whenever they did differ it would be possible in principle to construct a project that was beneficial for one and harmful for the other. We shall return to this issue at the end of Section IV.

[8]The meaning of "small" will become clear at the end of Section III.

[9]Returning to the discussion at the end of Section II it is now clear that since the dC_t terms will all have the same sign, the discount rate applied to them, once they were actually computed, would be irrelevant. Any discount rate would do.

[10]This conclusion continues to hold if the inequality in (6) is reversed, implying an isolation paradox of the opposite type from that considered by Marglin and Sen.

[11] Clearly, $V < 0$ would imply that W was lowered by an increase in each individual's lump sum income, Y^i.

[12] Another way of putting this is that N_ρ^x measures the net wealth generated by the project.

[13] The "shadow price of capital" analysis derives from the recognition that private savings will ordinarily be affected by the income changes resulting from a project, but confines itself to those resulting from the initial establishment of the project. On the other hand, the "social" rate of discount analysis discussed in Section II rests on the implicit assumption that the levels of private savings are unaffected by the income changes resulting from the adoption of a project, that all adjustment in response to a project takes place in consumption alone. This implies that the dC_t terms in (10) are the same as the B_t^x terms, but it is then not possible for the private savings equilibrium conditions (2) to hold both before and after the project is adopted. See Dasgupta, Marglin and Sen [1972], chapters 13 and 14.