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ON IMMISERIZING TRANSFERS AND IMMISERIZING GROWTH

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ON IMMISERIZING TRANSFERS AND IMMISERIZING GROWTH

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I. Introduction

In two recent papers in this JOURNAL, Chichilnisky (1980,1981) has argued that, in the context of models and under conditions stated by her, a transfer of resources from the North to the South is immiserizing to the South; and that increase in demand by the North of the South's exportable good will be immiserizing to the South, as well.

These assertions are utilised by the author to deduce policy conclusions concerning North-South trade relations, the optimal trade strategy for the South, and the question of the advisability of foreign aid. These are matters of considerable concern at the present juncture. It is essential therefore to examine the logical validity of these claims.

Both the subjects of immiserizing transfers and immiserizing growth are, of course, well understood by the students of international trade theory and statements of the conditions under which such "paradoxical" phenomena can be arrived at in specific models are readily available in that literature.^{2/} What we show here, however, is that such phenomena are derived incorrectly in the Chichilnisky papers; and that her results simply do not follow from the assumptions as stated by her. We do this by examining the Chichilnisky treatment of the transfer problem in Section 2, and her handling of the growth problem in Section 3. In view of the essential simplicity of the Chichilnisky models, it should be both sufficient and illuminating to utilise the simple geometric methods of trade theory to demonstrate, as we do, the mistakes in Chichilnisky. However, the Appendix provides a simple formal algebraic treatment of the Chichilnisky models and deduces the correct results.

II. Immiserizing Transfers from Abroad:

In Chichilnisky (1980), a simple exchange model with fixed coefficients in consumption is used. It is a so-called North-South model, with the North differentiated into Rich and Poor groups, and the South kept homogeneous; she endows each group with fixed quantities of 2 goods, A and B, and with fixed consumption coefficients. Besides, the South spends a larger proportion of income than the Poor on B goods whereas the latter, in turn, spend a greater proportion on B goods than their Rich compatriots. With the Rich transferring A goods to the South, the following theorem is stated (1980, p. 510):

Theorem 1: Assume that the endowments of the South are small, consisting mostly of basic goods B and that conditions (C.1) and (C.2) are satisfied.

Then a transfer of the luxury or investment good A from the resources of the high income group in the North to the South will necessarily decrease the welfare of the South and increase the welfare of the North, in a (Walrasian) stable market.^{3/}

Unfortunately, this cannot be correct: and the error is immediately evident. Thus, take Figure 1, where the South is represented. Let E_1 be the endowment. Then, p_1 is the initial price ratio, C_1 the consumption point so that South exports B goods and U^S the social welfare for (homogeneous) South. Fixed coefficients in consumption are shown. Let South now receive A goods of amount $E_1 E_2$. Now, as in standard Samuelsonian 2-state argumentation, we can deduce a rise in excess demand for B goods in the world markets as a result of the transfer, at constant terms of trade, if the Rich spend a lower proportion of income on B goods than South does, as assumed by the author (1980, p. 509). Given Walrasian market stability, therefore, the relative price of

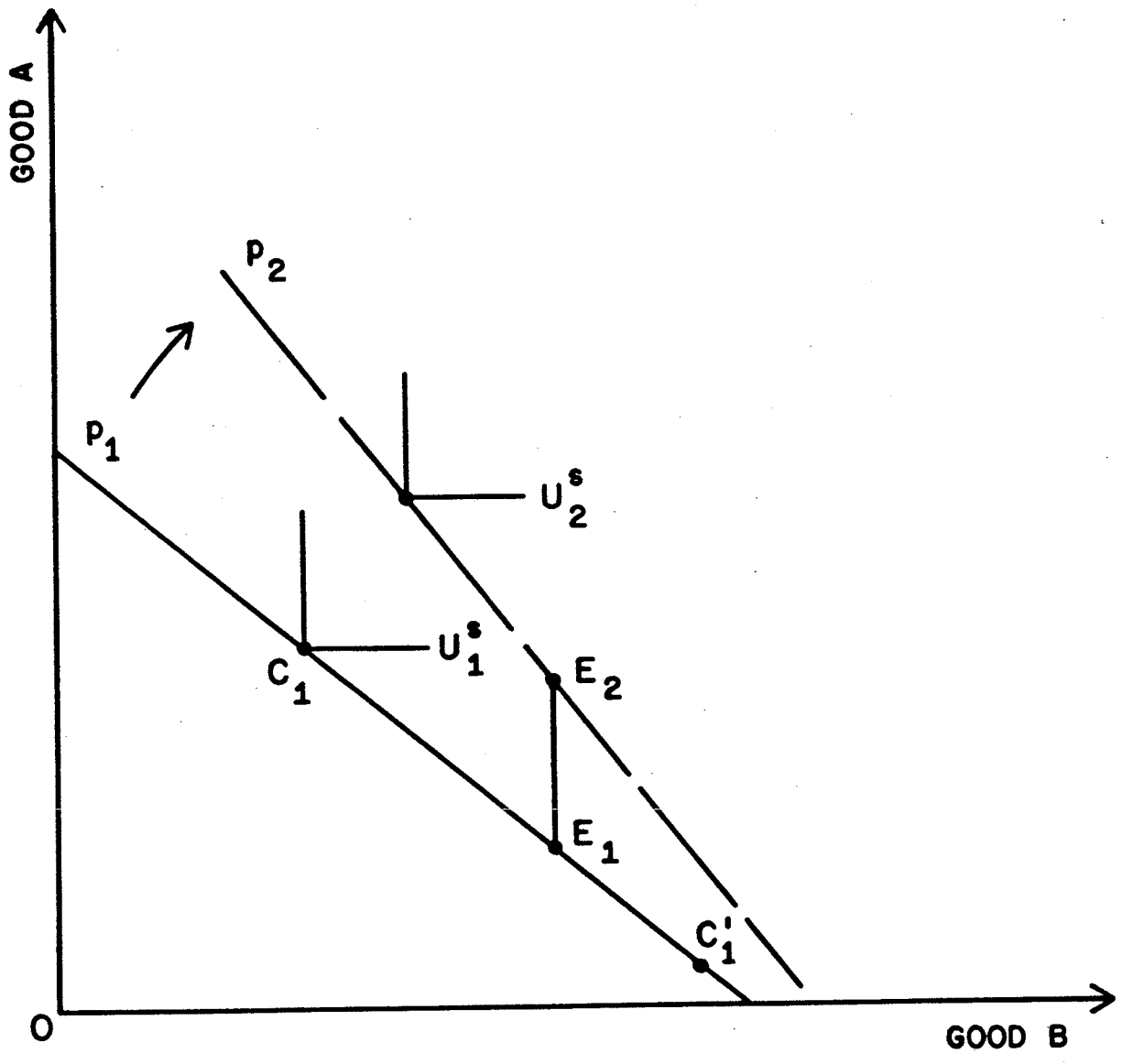


Figure 1

B goods must rise: p_1 yields to p_2 . Under these assumptions, which are totally consistent with the stated assumptions of Chichilnisky prior to, and including in (Theorem 1) paragraph 2, we have welfare improvement for the South; and hence the assertion that the transfer will necessarily decrease the welfare of the South is evidently invalid.

It is equally evident that we could show immiserization by assuming in Figure 1 that the South was exporting A goods rather than B goods: such that C_1 was replaced in Figure 1 by C'_1 (and a suitable shift in p_2). But then we have to reckon with the author's added assumption, in Theorem 1, paragraph 1, that "the endowments of the South are small, consisting mostly of basic goods B." If the endowment is sufficiently large in B goods and negligible in A goods, as is consistent with the assumption made, then the implied trade pattern initially would be definitely at a point such as C_1 with B goods being exported initially. In that case, welfare improvement of the South (rather than its immiserization) must follow!^{4/}

III. Immiserizing Growth:

The claims in Chichilnisky (1981) are unfortunately no more valid than those in Chichilnisky (1980). Before we discuss the model itself, we need to reproduce some of the author's statements in regard to the theory of immiserizing growth in Bhagwati (1958) (1968):

"Our results also differ both in assumptions and in policy conclusions from others in the existing formalised trade and growth literature on the immiserising effects of growth [cf. Bhagwati (1968, 1972)), Mundell (1968)]. In those works the results emerge from assumptions on international markets such as, for instance, different international elasticities of demand for the goods in which the North and the South specialise: the exports of the South are assumed to have inelastic demand internationally while the exports of the North have more elastic demands. Therefore, as the South attempts to grow more than the North, the prices of the exports of the South fall significantly, thus undermining its growth efforts...

The results in this paper have a dual character with respect to those of Bhagwati (1968, 1972), since ours depend more on the behaviour of supply of factors of production rather than on the elasticity of demand for goods.

(1981, p. 182)

It must be stated that Chichilnisky errs in regard to what Bhagwati (1958) showed as a condition for immiserizing growth in a country. He demonstrated that either an inelastic foreign offer curve, or ultra-biased growth with negative output-elasticity of supply of the importable good when the foreign offer curve is elastic, would make immiserizing growth possible. It is simply wrong to assert therefore that an inelastic foreign offer curve is necessary for immiserizing growth to occur in the Bhagwati case. The asserted "dual" character of Bhagwati's theorem with that apparently proved in Chichilnisky (1981), with the former depending on inelastic demand and the latter on factor supply, is thus incorrect.

But, apart from this error, there is also no appreciation of the fact that Bhagwati was dealing with domestic, exogenously-specified growth that immiserized the growing country. To explore immiserizing growth in her model, Chichilnisky would have to solve for the effect of expansion that is both domestic and exogenous, either due to technical change or due to capital accumulation, on domestic welfare: the way it is done, and needs to be done, in Bhagwati (1958) (1968) et al. This, she does not do.

Instead, as she is concerned with the effects on the South of assumed "shifts in the demand of the North" (1981, p. 178; footnote 11), she should be concerned

with the very different issue as to whether growth (or other parametric or policy shift) elsewhere can immiserize a country. And of course, it should come as no surprise that growth (or other shift) may imply an adverse shift in the foreign offer curve facing a country and therefore the country loses some of the gains from trade and is immiserized relative to the situation prior to this external growth (or other shift).

Unfortunately, however, even this analysis is erroneous in Chichilnisky (1981) because it is fatally flawed, as is the bulk of the paper, by the false argument that, in the model specified by her, an increase in the demand for the exportable (at each price) would reduce, rather than increase, the price of the exportable:

"Our case reflects, instead, shifts in the demand of the North, that increase the demand for the exportable at each price. This would under traditional assumptions increase the price of the exportable. In our case just the opposite effect takes place." (1981, p. 178, footnote 11)

The rest of this Section is therefore devoted to showing very simply using a geometric technique developed by Ronald Findlay, that this central proposition cannot hold in the model as specified by her; that, in fact, the model is extremely well-behaved indeed in this regard.^{5/} The model is, in essence, a 2x2x2 model with two points to note: the production functions are characterized by fixed coefficients, and the supplies of factors are variable with respect to rewards, in each country. Let the factors be K and L, and the goods be I and B. Then, the following holds for each of the 2 countries. In any incompletely specialized production equilibrium the goods price (P_I/P_B)

determines the factor price ratio (w/r) through the usual zero profit conditions under pure competition and constant returns to scale in production. These in turn determine factor quantities (K and L) through the postulated relationships between factor supplies and real factor rewards. Finally, given the factor supplies, outputs of I (Q_I) and B (Q_B) are determined using the condition that factors are fully employed.

Now, let P_I/P_B increase. We can then see that, if I is the K -intensive good, w/r falls, therefore K increases and L falls. Therefore, as in the argument underlying Rybczynski theorem, Q_I increases and Q_B falls. Therefore, given Walras' Law so that we concentrate on the I market, we see in Figure 2 that Q_I is a monotonically increasing function of P_I/P_B . As for demand for I , this is assumed constant. Therefore D_I is a vertical line. Now, add both countries to get aggregate D_I^A, Q_I^A curves, as in Figure 3.

One could not therefore get a stronger result; the equilibrium is unique and evidently Walras-stable. Now consider the North to have an increased demand for South's exportable good B , as in Chichilnisky. This is equivalent to the D_I^A curve shifting to the left to $D_I^{A'}$. We then get the orthodox conclusion that P_I/P_B must decrease with increased demand for the B good. Unfortunately, therefore, the Chichilnisky assertion to the contrary must be quietly buried. And, since this assertion is central to her paper, we must necessarily reject the theoretical and policy conclusions drawn in the paper as well.^{6/}

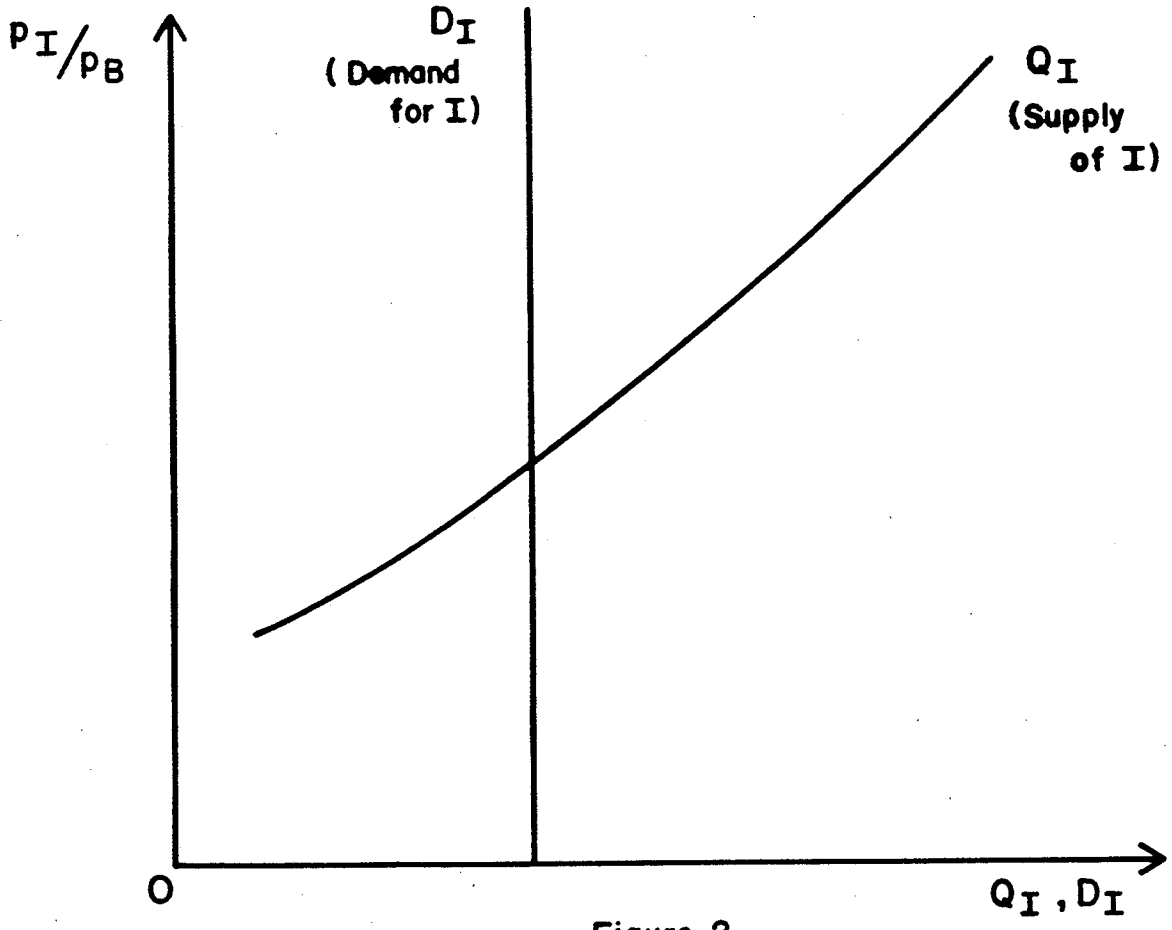


Figure 2

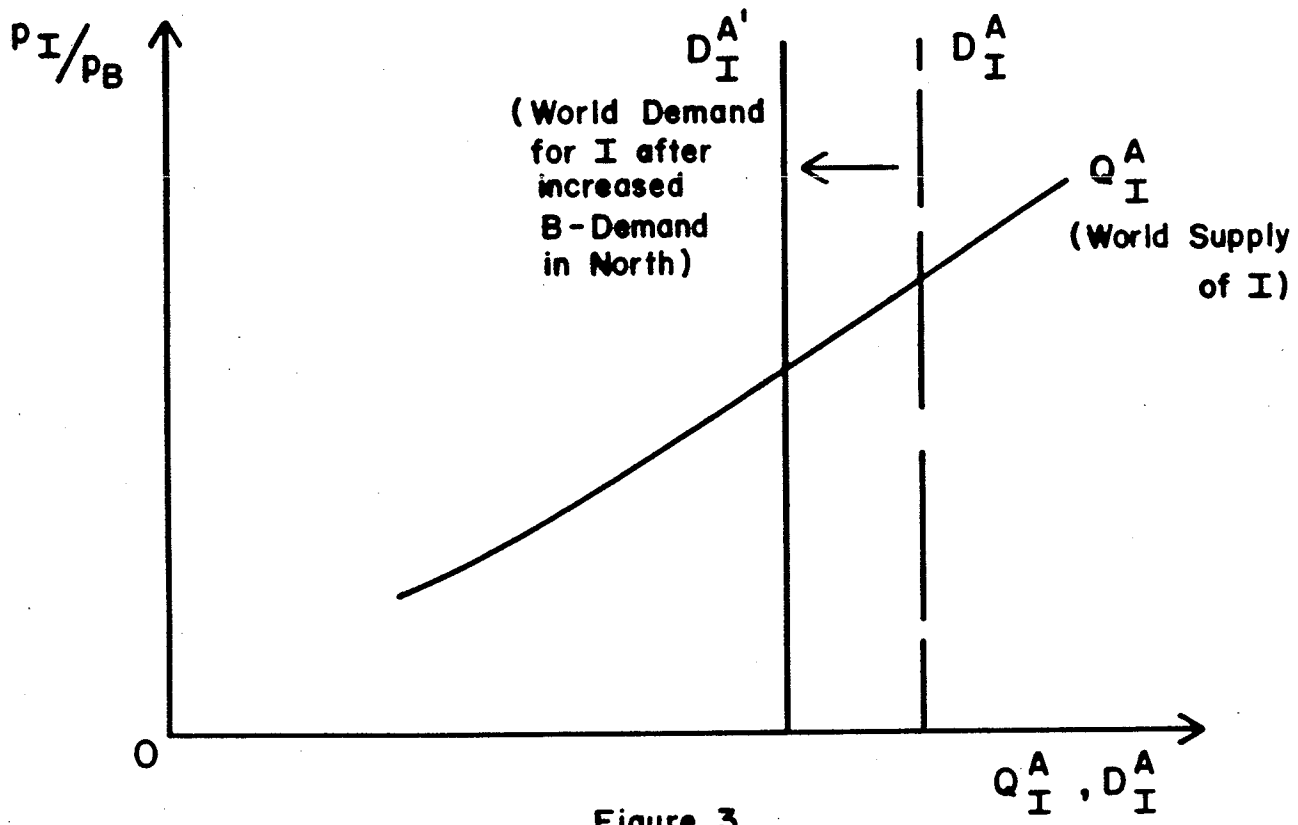


Figure 3

FOOTNOTES

1. Thanks are due to the National Science Foundation Grant No. SCS-8-25401 for support of Bhagwati's research underlying this paper. Conversations with Richard Brecher, Ronald Findlay, Tatsuo Hatta, Neantro Saavedra, and Pablo Serra have been very helpful.
2. To say that such paradoxes have been demonstrated - e.g. Bhagwati (1958, 1968) on immiserizing growth; Leontief (1936), Samuelson (1947, 1952), Brecher and Bhagwati (1981a, 1981b) on immiserizing transfers from abroad - is not to say that they are likely. Some of these results may be equally interpreted as showing the improbability of the paradoxical outcomes arising because of the conditions established for their occurrence.
3. Condition C.1 and C.2 ensure that an equilibrium exists with a positive price for basic goods and that equilibrium prices vary continuously with the parameters of the model.
4. If we were to assume, however, that the South was exporting A goods, the interesting question, of course, is whether we would then be able to show the South's immiserization, consistent with Walrasian stability. The answer to this question is that, yes, it is possible to show this in a 3-agent model even though, in the 2-agent Leontief (1936)-Samuelson (1947,1952) analysis, we know that Walrasian instability is required for the transferee to be immiserized. This proposition, developed in Brecher-Bhagwati (1981a) at

length and with conditions for immiserization carefully and simply spelled out, is easily understood in its essence as follows. Thus, recall the 2-country Samuelson-Mundell (1960) criterion for the welfare impact on the transferee: This is:

$$\begin{aligned} \frac{dU^{II}}{dT} &= 1 - \frac{m^I + m^{II} - 1}{\epsilon^I + \epsilon^{II} - 1} \\ &= \frac{\epsilon'^I + \epsilon'^{II}}{\epsilon^I + \epsilon^{II} - 1} \end{aligned} \tag{1}$$

where the transferor is country I, the transferee is country II, m^I and m^{II} are the marginal propensities to spend on importables (in a 2-good setting) and ϵ^I , ϵ^{II} are the compensated offer curve elasticities. Since ϵ'^I and ϵ'^{II} are definitely signed under the usual assumptions, $\frac{dU^{II}}{dT} > 0$ if $\epsilon^I + \epsilon^{II} > 1$, i.e. if market stability is assumed. The transferee cannot be immiserized.

When, however, an "outside" country or agent is assumed, say country III, which neither makes nor receives the transfer, then the formula must clearly be modified to take this into account. The income terms in the numerator will now belong only to the transferor I and the transferee II, whereas the offer curve elasticity ϵ^{II} must now be a weighted sum of the offer curve elasticities of countries II and III. Therefore $\frac{dU^{II}}{dT}$ will no longer show now simply the compensated elasticity terms in the numerator; and, depending on the relative patterns of trade of countries II and III, as also their respective marginal propensities to import, the immiserization of country II can arise, even though Walrasian stability obtains.

5. We are indebted to Ronald Findlay who demonstrated clearly the well-behaved nature of the Chichilnisky model and hence the error of her contrary assertions, by producing the simple argumentation we have used in the next. This error and several other problems afflicting the details of the Chichilnisky analysis, have been noted by Neantro Saavedra (1981) in a thorough comment.

6. We may stress that this is not to say that, in policymaking, we need not worry about possibilities of adverse outcomes for recipients of transfers such as foreign aid, for example. Not merely have trade theorists, for instance, produced interesting cases of such adverse outcomes, as we noted in footnote 2, using conventional value-theoretic general-equilibrium models, but also there is a substantial theoretical literature on issues such as the possibly deleterious effects of foreign aid via reduced domestic savings and domestic agricultural performance, and via destabilization efforts made possible through "aid-dependence," with some of this literature to be found in this JOURNAL itself (e.g. 1975, pp. 85-98)

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APPENDIX

(A) The Transfer Problem

Consider a country with the following Samuelsonian social utility function:

$$F(B,A) = \text{Min} [B, \lambda A], \lambda > 0 \quad (1)$$

and an endowment (E_B, E_A) of the two goods A and B. Then its excess demands (D_B, D_A) could be easily shown to be (with P^A, P^B as the price per unit of good A and B respectively in an accounting unit):

$$\text{For } (P^A = 0, P^B > 0) \quad D_B = 0 \quad D_A \geq \frac{1}{\lambda} E_B - E_A \quad (2a)$$

$$\text{" } (P^A > 0, P^B > 0) \quad D_B = \frac{P_A(\lambda E_A - E_B)}{P_A + \lambda P_B}, \quad D_A = \frac{P_B(E_B - \lambda E_A)}{P_A + \lambda P_B} \quad (2b)$$

$$\text{" } (P^A > 0, P^B = 0) \quad D_B \geq \lambda E_A - E_B, \quad D_A = 0 \quad (2c)$$

Now the indirect utility function $G(P^A, P^B, E_A, E_B)$ that corresponds to the direct utility function $F(B, A)$ is given by:

$$G(P^A, P^B, E_A, E_B) = E_B \quad \text{for } P_B > 0, P^A = 0 \quad (3a)$$

$$G(P^A, P^B, E_A, E_B) = \frac{\lambda [P_A E_A + P_B E_B]}{P_A + \lambda P_B} \quad \text{for } P_A > 0, P_B > 0 \quad (3b)$$

$$= \lambda E_A \quad \text{for } P^A > 0, P^B = 0 \quad (3c)$$

Now noting that in Chichilnisky (1980): (i) $\lambda = \frac{1}{a}$ for H, $\lambda = 1$ for L, and $\lambda = c$ for S (ii) $c > 1 > \frac{1}{a}$ (iii) H makes a transfer of T_A units of A and T_B units of B to S we get (denoting by E_j^i the net of transfer endowment of good j in country i):

$$E_A^H = H_A - T_A, \quad E_B^H = H_B - T_B \quad (4a)$$

$$E_A^L = L_A, \quad E_B^L = L_B \quad (4b)$$

$$E_A^S = S_A + T_A, \quad E_B^S = S_B + T_B \quad (4c)$$

The world excess demands D_A^W and D_B^W are (for $P^A = 0, P^B > 0$):

$$D_A^W \geq a(H_B - T_B) + L_B + \frac{1}{c}(S_B + T_B) - (H_A - T_A + L_A + S_A + T_A) \quad (5a)$$

$$D_B^W = 0 \quad (5b)$$

Hence if
$$a(H_B - T_B) + L_B + \frac{1}{c}(S_B + T_B) - (H_A + L_A + S_A) \leq 0 \quad (5c)$$

then $P^A = 0, P^B > 0$ for any $P^B > 0$ is a possible equilibrium price vector. Obviously, one can then choose good B as numéraire and set $P^B = 1$. Since (5c) does not involve T_A , if it holds for some T_B , it continues to hold as T_A is varied keeping T_B fixed. Hence changes in T_A do not affect equilibrium prices or welfare. From (3a) it can be easily shown that (as long as (5c) holds so that as T_B is varied equilibrium prices remain at

$P^A = 0, P^B = 1$), $\frac{dG^H}{dT_B} = -1, \frac{dG^L}{dT_B} = 0, \frac{dG^S}{dT_B} = 1$ so that the rich in the North lose and South gains by an increase in the transfer T_B .

Now for $P^A > 0, P^B = 0$,

$$D_A^W = 0 \quad (6a)$$

$$D_B^W \geq \frac{1}{a}(H_A - T_A) + L_A + c(S_A + T_A) - \{H_B - T_B + L_B + S_B + T_B\} \quad (6b)$$

Hence if
$$\frac{1}{a}(H_A - T_A) + L_A + c(S_A + T_A) - (H_B + L_B + S_B) \leq 0 \quad (6c)$$

then $P^A > 0$, $P^B = 0$ for any $P^A > 0$ is a possible equilibrium price vector. We can choose good A as numéraire and set $P^A = 1$. In this case, changes in T_B

with T_A fixed, do not affect (6c) and hence the equilibrium. As long as (6c), holds while we vary T_A , we get using (3c) that $\frac{dG^H}{dT_A} = -\frac{1}{a}$, $\frac{dG^L}{dT_A} = 0$, $\frac{dG^S}{dT_A} = c$.

Hence, once again the rich in the North lose and South gains by an increase in T_A .

Let us now consider equilibria in which $P^A > 0$ and $P^B > 0$ so that

$$D_A^W = \frac{P_B [H_B - T_B - \frac{1}{a} (H_A - T_A)]}{P_A + \frac{1}{a} P_B} + \frac{P_B (L_B - L_A)}{P_A + P_B} \quad (7a)$$

$$D_B^W = \frac{P_A [\frac{1}{a} (H_A - T_A) - (H_B - T_B)]}{P_A + \frac{1}{a} P_B} + \frac{P_A (L_A - L_B)}{P_A + P_B} \quad (7b)$$

$$+ \frac{P_B [S_B + T_B - c(S_A + T_A)]}{P_A + cP_B}$$

$$+ \frac{P_A [c(S_A + T_A) - (S_B + T_B)]}{P_A + cP_B}$$

Setting $P^B = 1$, defining $P = \frac{P^A}{P^B}$ and using Walras' law, the equilibrium

P is obtained from

$$D_B^W(P) \equiv P \frac{[H_A - T_A] - a(H_B - T_B)}{aP + 1} + \frac{P(L_A - L_B)}{P + 1} + \frac{P[c(S_A + T_A) - (S_B + T_B)]}{P + c} = 0 \quad (8)$$

Now $\frac{\partial D_B^W}{\partial T_A} = \frac{-P}{aP+1} + \frac{Pc}{P+c} > 0$ and $\frac{\partial D_B^W}{\partial T_B} = \frac{aP}{aP+1} - \frac{P}{P+c} > 0$ since $ca > 1$ (9)

$$\frac{\partial D_B^W}{\partial T_B} = \frac{aP}{aP+1} - \frac{P}{P+c} > 0 \quad \text{since } ca > 1 \quad (10)$$

Hence if $\frac{\partial D_B^W}{\partial P} > 0$ at an equilibrium P^* (i.e. if the equilibrium is

Walrasian Stable) then

$$\frac{\partial P^*}{\partial T_A} = \left(\begin{array}{c|c} -\frac{\partial D_B^W}{\partial T_A} & \frac{\partial D_B^W}{\partial P} \end{array} \right)_{P=P^*} < 0 \quad (11)$$

$$\frac{\partial P^*}{\partial T_B} = \left(\begin{array}{c|c} \frac{\partial D_B^W}{\partial T_B} & \frac{\partial D_B^W}{\partial P} \end{array} \right)_{P=P^*} < 0 \quad (12)$$

Hence an increase in transfers of either kind unambiguously reduce the equilibrium relative price of the A good.

Using (3b) we get:

$$\frac{\partial G^H}{\partial T_A} = \frac{-P^*}{(aP^*+1)} + \frac{[(H_A - T_A) - a(H_B - T_B)]}{(aP^* + 1)^2} \cdot \frac{\partial P^*}{\partial T_A} \quad (13)$$

$$\frac{\partial G^H}{\partial T_B} = \frac{-1}{(aP^*+1)} + \frac{[(H_A - T_A) - a(H_B - T_B)]}{(aP^* + 1)^2} \cdot \frac{\partial P^*}{\partial T_B} \quad (14)$$

Now $(H_A - T_A) - a(H_B - T_B)$ has the same sign as the excess demand for good B by H. Since $\frac{\partial P^*}{\partial T_A} < 0$ and $\frac{\partial P^*}{\partial T_B} < 0$, a sufficient though not necessary condition for both types of transfers to reduce the welfare of H unambiguously is that H is a net demander of basic goods. This is intuitively obvious also, since transfers raise the price of basic goods relative to luxuries and at unchanged prices, they reduce the expenditure of H. Hence, if H is a net demander of basic goods, both effects work with same direction to reduce welfare. Now,

$$\frac{\partial G^L}{\partial T_A} = \frac{L_A - L_B}{(P^* + 1)^2} \cdot \frac{\partial P^*}{\partial T_A} \quad (15)$$

$$\frac{\partial G^L}{\partial T_B} = \frac{L_A - L_B}{(P^* + 1)^2} \cdot \frac{\partial P^*}{\partial T_B} \quad (16)$$

Since L neither makes nor receives a transfer, its welfare is affected solely due to the change in equilibrium prices. As such it is made better (worse) off according as it is a net supplier(demander)of B goods i.e. according as $L_A - L_B < (>) 0$. Now,

$$\frac{\partial G^S}{\partial T_A} = \frac{cP^*}{(P^* + c)} + c \frac{\{c(S_A + T_A) - (S_B + T_B)\}}{(P^* + c)^2} \cdot \frac{\partial P^*}{\partial T_A} \quad (17)$$

$$\frac{\partial G^S}{\partial T_B} = \frac{c}{P^* + c} + c \frac{\{c(S_A + T_A) - (S_B + T_B)\}}{(P^* + c)^2} \cdot \frac{\partial P^*}{\partial T_B} \quad (18)$$

Hence S unambiguously benefits from either kind of transfer as long as he is a net supplier of B goods even after the transfers i.e. as long as $c(S_A + T_A) - (S_B + T_B) < 0$. In particular, if South's endowment (inclusive of transfer) of non-basic goods is small relative to its endowment (inclusive of transfer) of basic goods, it must necessarily gain by transfers.

It can be shown however that given $ca > 1$, in an equilibrium with $P^A > 0$, $P^B > 0$. $L_B \geq L_A$ implies $c(S_A + T_A) > (S_B + T_B)$ so that Northern poor and the South cannot both be net suppliers of B goods.

(B) Terms of Trade Problems

Following Chichilnisky (1981) let us denote the production functions for the South as:

$$B = \text{Min}\left[\frac{L^B}{a_1}, \frac{K^B}{c_1}\right] \quad (19a)$$

$$I = \text{Min}\left[\frac{L^I}{a_1}, \frac{K^I}{c_1}\right] \quad (19b)$$

with $D = a_1 c_2 - a_2 c_1 > 0$.

Denoting by W the wage rate, P_B the price of good B and R the rental rate on capital (all in terms of the I good as numéraire) the aggregate factor supply functions are:

$$L = \alpha\left(\frac{W}{P_B}\right) + \bar{L} \quad (20a)$$

$$K = \beta R + \bar{K} \quad (20b)$$

In any incompletely specialized production equilibrium profit maximization at positive and finite levels of output requires:

$$P_B = a_1 W + c_1 R \quad (21a)$$

$$1 = a_2 W + c_2 R \quad (21b)$$

Equation (21b) is the factor-price frontier of this model. We can rewrite equations (21a) and (21b) as:

$$W = (c_2 P^B - c_1)/D \quad (22a)$$

$$R = (a_1 - a_2 P^B)/D \quad (22b)$$

Clearly, with $D > 0$, $c_1 > 0$, $a_2 > 0$, a rise in P_B increases W and reduces R if the production equilibrium continues to be incompletely specialized. Also $\left(\frac{W}{P_B}\right)$, the real wage in terms of good B, also increases which in turn means (because of 20a and 20b) that a rise in P_B increases aggregate supply of labour and reduces aggregate supply of capital.

Now, given Chichilnisky's assumption that factor markets always clear, the outputs B^S, I^S of the South given P^B can be written as:

$$B^S(P^B) = \frac{c_2 L(P^B) - a_2 K(P^B)}{D} \quad (23a)$$

$$I^S(P^B) = \frac{a_1 K(P^B) - c_1 L(P^B)}{D} \quad (23b)$$

Since $L(P^B)$ is an increasing function of P^B and $K(P^B)$ is a decreasing function of P^B , $B^S(P^B)$ is an increasing function of P^B and $I^S(P^B)$ is a decreasing function of P^B . As such, given an exogenously specified demand \bar{I} for good I, if the economy is closed, equilibrium P^B is unique provided it exists (i.e. \bar{I} is a feasible demand). Chichilnisky states that there are two possible equilibrium P^B values for a given \bar{I} . This is plainly wrong: either there is none or there is only one.

Now in a trading equilibrium, if we denote the aggregate demand for South's output of B goods as $Q_{DB}(P^B, \theta)$ representing the sum of domestic demand $Q_D^S(P^B)$ and North's excess demand for South's exports i.e. $X_D^S(P^B, \theta)$ where θ is a shift parameter, equilibrium P^B is determined by

$$Q_{DB}(P^B, \theta) = B^S(P^B) \quad (24)$$

$$\text{Hence } \left(\frac{\partial Q_{DB}}{\partial P^B} - \frac{\partial B^S}{\partial P^B} \right) \cdot \frac{dP^B}{d\theta} = - \frac{\partial Q_{DB}}{\partial \theta} = - \frac{\partial X_D^S}{\partial \theta}$$

$$\text{Walrasian stability requires } \left(\frac{\partial Q_{DB}}{\partial P^B} - \frac{\partial B^S}{\partial P^B} \right) < 0 .$$

Hence, if there is a favourable shift in North's demand for South's exports

$$\frac{\partial X_D^S}{\partial \theta} > 0 \quad \text{then} \quad \frac{dP^B}{d\theta} > 0 . \quad \text{This in term means that } \frac{d}{d\theta} \left(\frac{W}{P^B} \right) > 0 \quad \text{as}$$

well, that is the real wages in the South in term of basic goods goes up as well.