

ECONOMIC GROWTH CENTER

YALE UNIVERSITY

P.O. Box 1987, Yale Station  
27 Hillhouse Avenue  
New Haven, Connecticut 06520

CENTER DISCUSSION PAPER NO. 448

OPTIMAL TRADE AND INDUSTRIAL POLICY UNDER OLIGOPOLY

Jonathan Eaton

and

Gene M. Grossman

December 1983

**Notes:** The authors acknowledges financial support of the National Science Foundation and the International Labor Affairs Bureau, U.S. Department of Labor.

Center Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the author to protect the tentative character of these papers.

## Optimal Trade and Industrial Policy Under Oligopoly

### ABSTRACT

In this paper we provide an integrative treatment of the welfare effects of trade and industrial policy under oligopoly, and characterize qualitatively the form that optimal intervention takes under a variety of assumptions about the number of firms, their conjectures about the response of their rivals to their actions, the substitutability of their products and the markets in which they are sold. We find that when no domestic consumption occurs optimal policy under duopoly with a single home firm depends on the difference between firms' actual responses to their rivals and the response that their rivals' conjecture. If conjectures are consistent, free trade is optimal. A tax or subsidy is indicated depending on the sign of the difference between the conjectured and the actual response. With more than one home firm but still no domestic consumption, an export tax is indicated if conjectures are consistent. Production subsidies and export tax-cum-subsidies can raise national welfare in the presence of domestic consumption, because these policies can mitigate the extent of the consumption distortion implicit in the deviation of price from marginal cost.

Jonathan Eaton  
Economic Growth Center  
Yale University  
Box 1987 Yale Station  
New Haven, CT 06520  
(203) 436-8414

Gene M. Grossman  
Woodrow Wilson School  
Princeton University  
Princeton, NJ 08544  
(609) 452-4823

## Optimal Trade and Industrial Policy Under Oligopoly\*

### I. Introduction

Implicit in many arguments for interventionist trade or industrial policy that have been advanced in the political realm appears to be an assumption that international markets are oligopolistic. It can be argued that international competition among firms in many industries is in fact imperfectly competitive, either because the number of firms is few, because products are differentiated, or because governments themselves have cartelized the national firms engaged in competition. They may do so implicitly through tax policy, or explicitly through marketing arrangements.

Government policies that affect the competitiveness of their firms in international markets, as well as the welfare of their consumers, involve not only traditional trade policy (trade taxes and subsidies) but policies that affect other aspects of firms' costs, such as output tax and subsidies. We refer to intervention of this sort as industrial policy.

Until recently the theory of commercial policy has considered the implications of intervention under conditions of perfect competition or, more rarely, pure monopoly. As a consequence, this literature cannot respond to many of the arguments that have been advanced in favor of activist government policies. The only argument for departing from laissez faire is the traditional optimal tariff argument. If (i) individual firms and consumers behave atomistically, (ii) the amount of trade that the economy engages in is sufficient to affect world prices, and (iii) the country's government can act as a Stackelberg leader in setting a trade tax before firms set prices or quantities, then a departure from free trade is optimal from the national perspective. From a world welfare perspective, however, free trade remains optimal. Our purpose in this paper is to extend the theory

---

\* We are grateful to Jim Brander and Avinash Dixit for helpful discussions.

of nationally optimal policy to situations in which individual firms exercise market power in world markets.

The primary implications of oligopoly for the design of trade policy are (i) that economic profits are not driven to zero, and (ii) that a price equal to marginal cost does not generally obtain. The first of these means that government policies that shift the industry equilibrium to the advantage of domestic firms may be socially beneficial from a national perspective. The second feature of oligopolistic competition suggests that trade policy may be a substitute for antitrust policy in an open-economy setting, if policies can be devised that effectively shrink the wedge between opportunity cost in production and marginal valuation to consumers.

A number of recent papers have focused on the profit-shifting motive for trade policy under oligopoly. Brander and Spencer (1982a) develop a model in which one home firm and one foreign firm produce perfectly substitutable goods, and compete in a third-country market. They consider a Cournot-Nash equilibrium, and find that if the home country's government can credibly pre-commit itself to pursue a particular trade policy before firms make production decisions (and if demand is not very convex), then an export subsidy is optimal.<sup>1</sup> Dixit (1984) has extended the Brander-Spencer result to cases with more than two firms, and establishes that an export subsidy in a Cournot oligopoly equilibrium is optimal as long as the number of domestic firms is not too large. Finally, Krugman (1983) shows that under increasing returns to scale, protection of a local firm in one market (e.g., by an import tariff) can shift the equilibrium to that firm's advantage in other markets by lowering its marginal cost of production.

These papers all provide examples in which interventionist trade policy can raise national welfare in imperfectly competitive environments. Yet each makes special assumptions about the form of oligopolistic competition, the substitutability of the goods produced and the markets in which they are sold. It is difficult to extract general principles for trade policy from this analysis. Our purpose here is to provide an integrative treatment of the welfare effects of trade and industrial policy under oligopoly, and to characterize the form that optimal intervention takes under a variety of assumptions about the number of firms, their assumptions about rivals' responses to their actions (their conjectural variations), the substitutability of their products and the countries where they are sold.

The paper is organized as follows. In the next section we consider a general conjectural variations model of a duopoly in which one home firm competes with one foreign firm either in the foreign firm's local market or in a third-country market. We find that the sign of the optimal trade or industrial policy (i.e. whether a tax or subsidy is optimal) depends on the relationship between the home firm's conjectural variations and the actual equilibrium reactions of the foreign firm. We note the form that optimal policy takes in Cournot and Bertrand equilibria and in what Bresnahan and Perry have called "consistent" conjectures equilibrium. When conjectures are consistent a policy of free trade is optimal.

In Section III we extend the analysis by expanding the number of firms while maintaining the assumption of no domestic consumption. Here we show that free trade is optimal under consistent conjectures if there is only one home firm, regardless of the number of foreign competitors. If two or more domestic firms compete in the foreign market, the optimal intervention in a symmetric consistent conjectures equilibrium is always to tax production

or exports.

Finally, in Section IV, we return to the duopoly case, and introduce domestic consumption. In the special case of perfectly substitutable products and consistent conjectural variations on the part of the home country firm, the introduction of a small production subsidy by the home country government raises national welfare. It does so by reducing the price for the commodity faced by domestic consumers, thereby reducing the difference between the marginal utility of consumption of the product and the cost of production, which is necessarily positive under oligopolistic competition. If, instead, trade intervention is considered, then either an export tax or a subsidy may be indicated.

The main findings of the paper are summarized in a concluding section.

## II. Optimal Trade Policy and the Role of Conjectural Variations: The Case of Duopoly

In this and subsequent sections we characterize optimal government policy in the presence of oligopolistic competition among domestic and foreign firms in international markets. Each firm produces a single product which may be a perfect or imperfect substitute for the output of its rivals. We specify competition among firms as Nash in output quantities with arbitrary conjectural variations.<sup>2</sup> The domestic government can tax (or subsidize) the output of domestic firms, tax (or subsidize) the exports of these firms, and tax (or subsidize) the imports from the foreign rivals of domestic firms. Its objective is to maximize national welfare.

The government acts as a Stackelberg leader vis-a-vis both domestic and foreign firms in setting ad valorem tax (subsidy) rates.<sup>3</sup> Thus firms set outputs taking tax and subsidy rates as given. In other words, the

government can pre-commit itself to a specific policy intervention that will not be altered even if it is sub-optimal ex post, once firms' outputs are determined. For simplicity we assume the absence of government policy in other countries, although this assumption has no qualitative implications for our results.

In this section we consider optimal government policy when oligopolistic competition takes its simplest possible form: a single domestic firm competes with a single foreign firm in a foreign market. In the absence of domestic consumption government trade policy (export taxes and subsidies) is equivalent to government industrial policy (output taxes and subsidies).

We assume that the government's objective function places equal weight on the home-firm's profit and government tax revenue. Its objective is therefore one of maximizing national product.

Denote the output (and exports) of the home firm by  $x$  and let  $c(x)$  be its total production cost,  $c'(x) > 0$ . Upper case letters denote corresponding magnitudes for the foreign firm, with  $C'(X) > 0$ . Pretax revenue of the home and foreign firms are given by the functions  $r(x, X)$  and  $R(x, X)$  respectively. These satisfy the conditions that

$$r_2(x, X) \equiv \frac{\partial r(x, X)}{\partial X} \leq 0$$

$$R_1(x, X) \equiv \frac{\partial R(x, X)}{\partial x} \leq 0$$

These conditions state that an increase in the output of the competing product lowers the total revenue of each firm. They are implied by the assumption that the products are substitutes in consumption. Total after-tax

profits of the home and foreign firms are given by:

$$\pi = (1 - t) r(x, X) - c(x)$$

and

$$\Pi = R(x, X) - C(X)$$

respectively. Here  $t$  denotes the ad valorem output (or export) tax. The domestic firm's conjecture about the foreign firm's output response to changes in its own output is given by the parameter  $\gamma$ . The foreign firm's corresponding conjectural variation is  $\Gamma$ .

The Nash equilibrium quantities, given the level of home country policy intervention, are determined by the first-order conditions

$$(1 - t)[r_1(x, X) + \gamma r_2(x, X)] - c'(x) = 0 \quad (1)$$

$$R_2(x, X) + \Gamma R_1(x, X) - C'(X) = 0 \quad (2)$$

We assume that the second-order conditions for profit maximization are satisfied.

We now demonstrate:

Theorem 1: A positive (negative) output or export tax can yield higher national welfare than laissez-faire ( $t = 0$ ) if the home firm conjectures a foreign change in output in response to an increase in its own output that is smaller (larger) than the actual response.

Proof: National welfare generated by the home firm is given by  $w$  where

$$\begin{aligned} w &= (1 - t)r(x, X) - c(x) + tr(x, X) \\ &= r(x, X) - c(x) \end{aligned} \quad (3)$$

The change in welfare resulting from a small change in the tax (or subsidy) rate  $t$  is<sup>4</sup>

$$\frac{dw}{dt} = [r_1(x, X) - c'(x)] \frac{dx}{dt} + r_2(x, X) \frac{dX}{dt} \quad (4)$$

Substituting the first-order condition, (1), into (4), we obtain<sup>5</sup>

$$\frac{dw}{dt} = [-\gamma r_2 - \frac{tc'}{1-t}] \left(\frac{dx}{dt}\right) + r_2 \left(\frac{dX}{dt}\right) \quad (5)$$

Expression (2) implicitly defines the output of the foreign firm,  $X$ , as a function of domestic output  $x$ . Denote this function  $\psi(x)$ . The tax rate  $t$  does not appear directly as an argument of this function, since  $t$  does not appear in expression (2). Therefore  $dX/dt = \psi'(x)(dx/dt)$ . Define  $g = (dX/dt)/(dx/dt) = \psi'(x)$ ; the term  $g$  measures the slope of the foreign firm's actual reaction to  $x$ . A first-order condition for maximizing national welfare obtains when  $dw/dt = 0$ , or, incorporating the definition of  $g$  into equation (5)

$$-r_2(g - \gamma) = tc'/(1 - t) \quad (6)$$

Since  $r_2 < 0$ , the left-hand and right-hand sides of expression (6) are of the same sign if  $1 > t > 0$  and  $g > \gamma$ , or  $t < 0$  and  $g < \gamma$ . The term  $g - \gamma$  is the difference between the actual response of  $X$  to a change in  $x$  (i.e.,  $\psi'(x)$ ) and the home firm's conjectural variation. When  $g > \gamma$  a tax can yield more income than laissez-faire, conversely when  $g < \gamma$ .

Q.E.D.

We now turn to some specific conjectural variations that are commonly assumed in models of oligopolistic competition.

#### A. Cournot Conjectures

Under Cournot behavior, each firm conjectures that when it changes its output the other firm will hold its output fixed. Thus,  $\gamma = \Gamma = 0$  in this case, and (6) becomes

$$-gr_2 = tc'/(1-t) \quad (7)$$

Totally differentiating the equilibrium conditions (1) and (2) to solve for  $g$  this expression may be written:

$$\frac{r_2 R_{21}}{R_{22} - C''} = \frac{tc'}{1-t} \quad (8)$$

The second-order condition for the foreign firm's profit maximization ensures that the left-hand side of this expression has the sign of  $R_{21}$ . Letting  $t^*$  denote the optimal export tax (or subsidy, if negative), we have established

Proposition 1: In a Cournot duopoly with no home consumption,  $\text{sgn } t^* = \text{sgn } R_{21}$ .

In the case of Cournot duopoly  $R_{21}$  is the slope of the foreign firm's reaction curve. If  $R_{21} < 0$ , then an increase in home output causes the foreign firm to reduce its output. Linear demand necessarily implies  $R_{21} < 0$ , and many, but not all, specifications of demand imply this sign as well.

Proposition 1 constitutes a slight generalization of the Brander-Spencer (1982a) argument for an export subsidy to situations in which the

competing firms produce imperfect substitutes: an export subsidy raises domestic welfare in a Cournot equilibrium by transferring industry profit to the domestic firm. Graphically, the tax shifts the domestic reaction curve so that it intersects the foreign curve at the point that the home firm would have chosen had the home firm been a Stackelberg leader.

In the case of Cournot conjectures the home firm's conjecture that the foreign firm will not lower its output in response to its own quantity increase is an overly pessimistic one. When  $R_{21} < 0$ , the foreign firm's actual reaction to such a deviation is to reduce output. The home government, with its ability to pre-commit itself to a tax on output before firms determine their output levels, can take advantage of the difference between the firm's conjecture and the true response.

#### B. Bertrand Conjectures

In a Bertrand equilibrium, each firm conjectures that its rival will hold its price fixed in response to any changes in its own price. Define the direct demand functions for the output of the home and foreign firms as  $d(p, P)$  and  $D(p, P)$  respectively. The total profits of the two firms are therefore

$$\pi(p, P) = (1-t)pd(p, P) - c(d(p, P))$$

$$\Pi(p, P) = PD(p, P) - C(D(p, P)) .$$

Each firm sets its price to maximize its profit taking the other firm's price as constant. First-order conditions for a maximum imply

$$\pi_1 = (1 - t)(d + pd_1) - c'd_1 = 0 \quad (9a)$$

$$\Pi_2 = D + (P - C')D_2 = 0 \quad (9b)$$

While the home firm conjectures that the foreign firm will not change its price in response to a change in  $p$ , the true response is given, from differentiating (9b), by

$$\frac{dP}{dp} = \frac{\Pi_{21}}{\Pi_{22}} = \frac{D_1 + (P - C')D_{21} - C''D_1D_2}{- [2D_2 + (P - C')D_{22} - C''(D_2)^2]} \quad (10)$$

The second-order condition for the foreign firm's profit maximization implies that the sign of expression (10) is the same as that of  $\Pi_{21}$ . If the two products are substitutes (i.e.  $d_2 > 0$  and  $D_1 > 0$ ) and returns to scale are non-increasing ( $c'' \geq 0$ ,  $C'' \geq 0$ ) a positive response will emerge unless an increase in its rival's price has a significantly negative effect on the slope of the demand curve facing the home firm. In the special cases of either perfect substitutes or linear demands a positive response necessarily obtains. There is consequently a presumption in the case of competition between producers of goods that are substitutes that the Bertrand conjecture on the part of a firm that is cutting its price to expand its sales is overly optimistic.

The actual and conjectured price responses can be translated into quantity responses by totally differentiating the demand functions to obtain

$$\begin{bmatrix} dx \\ dX \end{bmatrix} = \begin{bmatrix} d_1 & d_2 \\ D_1 & D_2 \end{bmatrix} \begin{bmatrix} dp \\ dP \end{bmatrix}$$

The Bertrand conjecture on the part of the home firm implies a conjectured quantity response given by

$$\gamma = \left( \frac{dX}{dp} / \frac{dx}{dp} \right) \Big|_{dP=0} = D_1/d_1 \quad (11)$$

The actual response is

$$g = \frac{dX}{dp} / \frac{dx}{dp} = \frac{D_1 - D_2 \Pi_{21} / \Pi_{22}}{d_1 - d_2 \Pi_{21} / \Pi_{22}} \quad (12)$$

The term  $g - \gamma$  is positive as long as  $\Pi_{21} > 0$  (the foreign firm responds to a price cut by cutting its price). Applying theorem 1 we conclude:

Proposition 2: In a Bertrand duopoly with no home consumption  $\text{sgn } t^* = \text{sgn } \Pi_{21}$ .

Presumption regarding the sign of the optimal trade intervention when duopolistic behavior is Bertrand is exactly the opposite of that in the Cournot case; that is, an export tax is generally required. The intuition for this result is instructive. When a firm holds a Bertrand conjecture its belief about its rival's reaction to its own output expansion is typically overly optimistic. It conjectures that the competitor will respond to its own price cut (output expansion) by maintaining a constant price, whereas for most demand and cost structures (including the cases of perfect substitutes and of linear demands) the equilibrium response of the competitor is to lower its price. (Note the contrast with the Cournot case.) Thus, the government can shift industry profit to the home firm by forcing it to be less aggressive so as to take into account the true slope of the foreign response curve, that is by taxing sales.

### C. Consistent Conjectures

The final special case we consider is one in which duopolistic behavior is characterized by consistent conjectures. A consistent conjectures equilibrium (CCE), as defined and analyzed by Bresnahan (1981) and Perry (1982), among others, is one in which each firm's conjectural variations are equal to the actual equilibrium responses of its rivals that would result if that firm actually were to change its output by a small amount. Bresnahan (1981, p. 942) argues that consistency of conjectures is a reasonable restriction to place on oligopolistic behavior if exogenous changes in the market environment are frequent enough to allow firms to learn their rival's true responses. In our case, for example, changes in trade policy or in factor prices in one country would shift a single firm's reaction curve, and the locus of new equilibria would provide the firm with information about the slope of its rival's reaction curve.

The slope of the foreign reaction curve in our model is given by  $(dX/dt)/(dx/dt) = g$ . Thus, a consistent conjectures equilibrium is defined by  $\gamma = g$ . The following proposition follows immediately from expression (6):<sup>6</sup>

Proposition 3: In a duopoly with consistent conjectures and no home consumption,  $t^* = 0$ .

The optimality of free trade under consistent conjectures emerges because there exists no shift of the home firm's reaction curve that can transfer industry profit to that firm, given the response of its rival.

\* \* \*

The duopoly example with no home consumption highlights the profit-shifting motive for trade policy intervention in an imperfectly competitive

market. We have endowed the home government with a strategically advantageous position in relation to firms by assigning it the role of a Stackelberg leader in setting policy. In such circumstances the home government can raise national product by shifting the duopoly equilibrium to exploit any deviation of the home firm's conjectures from the actual equilibrium response of the foreign firm. If the home firm is overly pessimistic in its conjecture about the reaction to an increase in its own output an export subsidy raises income, while if its conjecture is too optimistic an export tax raises income. When conjectures are actually "correct," as they are in a consistent conjectures equilibrium, then no scope remains for shifting profit to the home firm by shifting its reaction function, and free trade is optimal from the national perspective.

### III. Optimal Trade Policy: The Case of Multi-Firm Oligopoly and Consistent Conjectures

In this section we extend our analysis by introducing more than two firms. For analytical convenience we confine our attention to symmetric configurations. We continue to assume throughout this section that there is no home country consumption of the outputs of the oligopolistic industry.

Suppose there are  $n$  home firms and  $m$  foreign firms in the industry.

The profit of home firm  $i$  is

$$\pi^i = (1 - t^i)r^i(x^1, \dots, x^n, X^{n+1}, \dots, X^{n+m}) - c^i(x^i).$$

while a foreign firm earns

$$\Pi^j = R^j(x^1, \dots, x^n, X^{n+1}, \dots, X^{n+m}) - c^j(X^j),$$

where  $t^i$  denotes the output tax imposed on firm  $i$ . The conjecture of firm  $i$  about the response of the  $j$ th firm's output to a change in its own output is denoted  $\gamma^{ij}$ ,  $j \neq i$ ,  $j = 1, \dots, n+m$ .

The home country national product derived from this industry is

$$w = \sum_{i=1}^n r^i - c^i$$

First-order conditions for profit maximization are:

$$(1 - t^i)r_i^i - c^{i'} + (1 - t^i) \sum_{\substack{j=1 \\ j \neq i}}^{n+m} r_j^i \gamma^{ij} = 0; \quad (i = 1, \dots, n) \quad (13a)$$

$$R_j^j - c^{j'} + \sum_{\substack{i=1 \\ i \neq j}}^{n+m} R_j^i \Gamma^{ij} = 0; \quad (j = n+1, \dots, n+m) \quad (13b)$$

Imposing the symmetry assumption and totally differentiating (13a) and (13b) at  $t^i = 0$  ( $i = 1, \dots, n$ ), we obtain

$$A^{(n+m)} \left[ \frac{dx}{dX} \right] = \left[ \frac{dt}{0} \right]^{(n)} \lambda$$

where  $A^{(r)}$  is an  $r \times r$  matrix with diagonal elements  $\alpha$  and off-diagonal elements  $\beta$ ,  $dx$  is an  $n$ -dimensional column vector with elements  $dx^i$ ,  $dX$  is an  $m$ -dimensional column vector with elements  $dX^j$ ,  $dt^{(n)}$  is an  $n$ -dimensional column vector with elements  $dt^i$  and  $0^{(m)}$  is an  $m$ -dimensional column vector of 0's. We have defined:<sup>7</sup>

$$\alpha \equiv r_{11}^1 - c^{1''} + (n+m-1)r_{12}^1 \gamma$$

$$\beta \equiv r_{12}^1 + r_{22}^1 \gamma + (n+m-2)r_{32}^1 \gamma$$

$$\lambda \equiv r_1^1 + (n+m-1)r_2^1 \gamma$$

and  $\gamma \equiv \gamma^{1j}$ . In considering an export tax we set  $t^i = t$ ,  $i = 1, \dots, n$ . Differentiating home country national product with respect to the vector  $t$  at  $t = 0$  yields:

$$\begin{aligned} \left. \frac{dw}{dt} \right|_{t=0} &= nr_2^1 [(n-1) - (n+m-1)\gamma] (dx^2/dt) + r_2^{1nm} (dX^{n+1}/dt) \\ &= nr_2^1 (dx^2/dt) \{ (n-1)(1-\gamma) + m \left[ \frac{dX^{n+1}/dt}{dx^2/dt} - \gamma \right] \} \end{aligned} \quad (15)$$

In this section we focus on the case in which firms form conjectures consistently. As we showed in section II, when there is a single domestic firm and a single foreign firm and conjectures are consistent, national welfare is maximized under laissez-faire. More generally, the direction of departure from laissez-faire depends upon the sign of the difference between the actual and the conjectured response of the foreign firm. By considering the case of consistent conjectures we can isolate the effect of increasing the number of firms on optimal policy.

Using (15), we are able to demonstrate:

Proposition 4: In a symmetric, oligopolistic, consistent conjectures equilibrium with  $n$  home firms,  $m$  foreign firms, and no home consumption, the optimal production (export) tax is zero if  $n=1$  and positive if  $n>1$ .

The proof of this proposition is provided in the appendix. It can be understood intuitively by noting that when conjectures are consistent,

the profit-shifting motive for government intervention disappears. What remains is the standard optimal-tariff prescription. Whenever there is more than a single home country firm, each home firm ignores the pecuniary externality it imposes on other domestic firms when it raises its output. Private incentives lead to socially excessive outputs, since home income includes all home firm profits. The government can enforce the cooperative equilibrium in which home firms act as a group to maximize the home country's total profit by taxing exports or sales. This externality does not arise when there is only one home firm; consequently, free trade is optimal in that case.

If home firms conjecture that their rivals react less aggressively than they actually do, as is often the case in Bertrand equilibrium, then the desirability of an output or export tax is enhanced. Conversely, when domestic firms conjecture responses that are more aggressive than the actual ones, then either a tax or a subsidy may raise national product, depending upon whether the national-market-power effect or the profit-shifting effect of the policy dominates.

#### IV. Trade and Industrial Policy in the Presence of Domestic Consumption

Thus far we have ruled out domestic consumption of the outputs of the oligopolistic industry under consideration. This has allowed us to focus on the profit-shifting motive for trade policy. However, by making this assumption, we have neglected a second way in which interventionist trade or industrial policy might yield welfare gains when markets are imperfectly competitive. Since oligopolistic markets are generally characterized by a difference between the price and marginal cost of a product, there is a potential second-best role for trade and industrial

policy (in the absence of first-best antitrust policy) to reduce this distortion.

When domestic consumption is positive, production taxes or subsidies and export taxes or subsidies are no longer identical. In this section we will consider the welfare effects of both types of policies in the duopoly model of Section II, recognizing that when there is more than one domestic firm, the national-market-power motive for taxation of output or exports is always present. In addition, in order to focus on the considerations for trade and industrial policy introduced by the presence of domestic consumption, we shall examine only the consistent-conjectures duopoly. Recall that in this case free trade is optimal when there is no domestic consumption.

To make our point as simply as possible, we assume that the duopolistic competitors produce a single, homogeneous good. We also assume perfect arbitrage with zero transport costs, so that under a production tax or subsidy consumers at home and abroad face the same price for the product. In other words, firms cannot price discriminate by setting different prices in different countries.

#### A. Production Tax or Subsidy

Let  $p(x+X)$  be the inverse world demand function and let home country direct demand be  $h(p)$ . The corresponding foreign demand is  $H(p)$ . If a production tax at rate  $t$  is imposed the profit of the domestic firm is  $\pi = (1 - t)p(x + X)x - c(x)$ . Consumer surplus at home is  $\int_p^\infty h(q) dq$ .<sup>8</sup> Domestic tax revenue is  $tpx$ . Summing these gives total home country welfare from producing, consuming and taxing the product:

$$w = px - c + \int_p^{\infty} h(q) dq$$

The change in home welfare resulting from a small change in the output tax is

$$\frac{dw}{dt} = (p + xp' - c') \frac{dx}{dt} + xp' \frac{dX}{dt} - h \frac{dp}{dt}$$

Substituting the first-order condition for the home-firm's profit maximization, this becomes:

$$\frac{dw}{dt} = \{xp'(g - \gamma) + t[p + xp'(1 + \gamma)]\} \frac{dx}{dt} - h \frac{dp}{dt} \quad (16)$$

Evaluating (16) at  $t = 0$ , and imposing the condition that conjectures are consistent ( $g = \gamma$ ), we find  $dw/dt = -hdp/dt$ . The choice between a production tax and a production subsidy hinges on which policy would lower the price faced by domestic consumers, thereby reducing the consumption distortion associated with imperfect competition.

It is easy to calculate  $dp/dt = p'(dx + dX)/dt$ . Applying Cramer's rule to the total differentials of the two firms' first-order conditions, we have

$$\frac{d(x + X)}{dt} = \frac{c'}{\Delta} [(C' - p)X - C''] \quad (17)$$

where  $\Delta$  is the determinant of the 2x2 Jacobian matrix, which is assumed to be positive for stability. If foreign marginal cost is increasing ( $C'' > 0$ ), then  $p > C'$ , and the right hand side of (17) is unambiguously negative.

A production subsidy raises world output, and hence lowers world price. Alternatively, if marginal costs at home and abroad are constant ( $c'' = 0$  and  $C'' = 0$ ), then the consistent conjectures equilibrium is the Bertrand equilibrium (see Bresnahan, 1981), so that  $p = C'$  and  $d(x + X)/dt = 0$ . In this case the optimal industrial policy is laissez faire.

Proposition 5: In a homogenous product duopoly with consistent conjectures and non-zero domestic consumption,

- (i) if  $c'' = 0$  and  $C'' = 0$ , then  $t^* = 0$
- (ii) if  $C'' > 0$ , then  $t^* < 0$

#### B. Export Tax or Subsidy

Finally, we consider the welfare effects of a small export tax at rate  $\tau$ . Under this policy domestic consumers pay a price  $p(1-\tau)$  for the good, and home government revenue is  $\tau(x - h)$ . The world inverse demand function is now written as  $p(x + X, \tau)$ , where

$$p_1 = 1/\{H'(p) + (1 - \tau)h'[p(1 - \tau)]\} \quad \text{and} \quad p_2 = ph'[p(1 - \tau)]p_1.$$

Proceeding as before, we find

$$\left. \frac{dw}{d\tau} \right|_{\tau=0} = hp_1 \frac{d(x + X)}{d\tau} + p_2(x - h)$$

In this case, however, it is no longer possible to sign unambiguously the effect of a small tax or subsidy on total world output. In addition, there is a second term that now enters the expression for  $dw/d\tau$ ,

which at  $\tau = 0$  is unambiguously positive or negative depending upon whether the home country is a net exporter or importer of the product. Given total output, an export tax raises the world price of the export good since it subsidizes domestic consumption. This standard terms-of-trade effect favors an export tax or import tariff, just as it does when the market is competitive.

To recapitulate the argument of this section, either an export tax or an export subsidy may raise domestic welfare in a duopolistic market with domestic consumption. When conjectures are consistent, any profit-shifting motive for policy intervention is eliminated. What remains is a standard terms-of-trade effect, and what might be termed a "consumption-distortion effect," arising from the gap between price and marginal cost. The former always indicates an export tax or import tariff, while the latter may favor either a tax or a subsidy, depending on the precise forms of the demand and cost functions.

## V. Conclusions

We have analyzed the welfare effects of trade policy and industrial policy (production taxes and subsidies) for a range of specifications of an oligopolistic industry. A number of general propositions for optimal policy emerge. First, either trade policy or industrial policy may raise domestic welfare if oligopolistic profits can be shifted to home country firms. Policies that achieve this profit shifting can work only if the government is able to set its policy in advance of firms' production decisions, and if government policy commitments are credible. Furthermore, in the duopoly case profits can be shifted only if firms' conjectural variations differ from the true equilibrium responses that would result if

they were to alter their output levels. The choice between a tax and a subsidy in this case depends on whether firms' conjectures about their rival's response are overly optimistic or overly pessimistic.

Second, whenever there is more than one domestic firm, competition among them is detrimental to home country social welfare. In other words, there exists a pecuniary externality since each domestic firm does not take into account the effect of its own actions on the profits of other domestic competitors. A production or export tax will lead domestic firms to restrict their outputs, shifting them closer to the level that would result with collusion. In this familiar way a production or export tax enables the home country to exploit its monopoly power in trade fully.

Finally, when there is domestic consumption of the output of the oligopolistic industry, there are two further motives for policy intervention. First, consumers' marginal valuation of the product will generally differ from domestic marginal cost of production due to the collective exertion of monopoly power by firms in the industry. A welfare improving policy for this reason should increase domestic consumption. When industrial policy is used, a production subsidy will achieve this result, whereas the appropriate trade policy instrument may be either an export (or import) tax or an export (or import) subsidy. Second, there is the usual externality caused by the multiplicity of small domestic consumers, who do not take into account the effect of their demands on world prices. Industrial policy cannot be used to overcome this externality, but if the country is a net exporter (importer) an export (import) tax will have a favorable impact on the country's terms of trade. The formulation of optimal trade or industry policy requires the weighting of these various influences.

Appendix

In this appendix we prove Proposition 4. To do so, we first prove the following lemma:

Lemma: Let  $A^{(r)}$  be an  $r \times r$  matrix with diagonal elements  $\alpha$  and other elements  $\beta$ . Let  $B^{(r)}$  be the matrix formed by deleting the first row and second column of  $A^{(r+1)}$ . Then

$$|A^{(r)}| = (\alpha - \beta)^{r-1} [\alpha + \beta(r - 1)]$$

$$|B^{(r)}| = \beta(\alpha - \beta)^{r-1}$$

Proof: The proof is by induction. The formulae hold trivially for  $r = 1$ , since  $|A^{(1)}| = \alpha$  and  $|B^{(1)}| = \beta$ .

Now suppose

$$|A^{(r-1)}| = (\alpha - \beta)^{r-2} [\alpha + (r - 2)\beta], \quad r > 2 \tag{A1}$$

and

$$|B^{(r-1)}| = \beta(\alpha - \beta)^{r-2}, \quad r > 2 \tag{A2}$$

Expanding  $|A^{(r)}|$  along its first row, we have

$$|A^{(r)}| = \alpha |A^{(r-1)}| - (r - 1)\beta |B^{(r-1)}| \tag{A3}$$

Similarly expanding  $|B^{(r)}|$  along its first row yields

$$|B^{(r)}| = \beta |A^{(r-1)}| - (r - 1)\beta |B^{(r-1)}| \tag{A4}$$

Substituting (A1) and (A2) into (A3), we have

$$\begin{aligned} |A^{(r)}| &= \alpha(\alpha - \beta)^{r-2}[\alpha + (r - 2)\beta] - \beta^2(r - 1)(\alpha - \beta)^{r-2} \\ &= (\alpha - \beta)^{r-2}[\alpha + \beta(r - 2)\alpha\beta - (r - 1)\beta^2] \\ &= (\alpha - \beta)^{r-1}[\alpha + (r - 1)\beta] \end{aligned}$$

A similar substitution into (A4) yields

$$\begin{aligned} |B(r)| &= \beta(\alpha - \beta)^{r-2}[\alpha + (r - 2)\beta] - (r - 1)\beta^2(\alpha - \beta)^{r-2} \\ &= \beta(\alpha - \beta)^{r-2}[\alpha + (r - 2)\beta - (r - 1)\beta] \\ &= \beta(\alpha - \beta)^{r-1} \end{aligned}$$

Q.E.D.

We are now able to prove Proposition 4, which we restate here for convenience.

Proposition 4: In a symmetric, oligopolistic, consistent conjectures equilibrium with  $n$  home firms,  $m$  foreign firms, and no home consumption, the optimal production (export) tax is zero if  $n=1$  and positive if  $n > 1$ .

Proof

In the case of consistent conjectures firm  $i$  anticipates a response on the part of firm  $j$  to an exogenous change in its own output that corresponds to its actual general equilibrium response. To generate an exogenous change in  $x^i$  consider the effect of variation in the tax  $t^i$  on the output of firm  $i$ . Such a variation affects only firm  $i$ 's first-order condition for profit maximization, given output of all other firms. The total response in the output of any other firm to the

variation in  $t^i$  derives solely from the variation it induces in  $x^i$ .  
Therefore, the consistent conjecture of firm  $i$  about firm  $j$ 's response to a change in its own output is given

$$\gamma = \begin{cases} (dx^j/dt^i)/(dx^i/dt^i) & \text{for } j = 1, \dots, n \text{ } j \neq i \\ (dX^j/dt^i)/(dx^i/dt^i) & \text{for } j = n+1, \dots, n+m \end{cases}$$

where

$$\frac{dx^i}{dt^i} = \lambda |A^{(n+m-1)}| / |A^{(n+m)}|$$

$$\frac{dx^j}{dt^i} \Big|_{t=0} = \frac{dX^k}{dt^i} \Big|_{t=0} = -\lambda |B^{(n+m-1)}| / |A^{(n+m)}| \quad \text{for } j=1, \dots, n; j \neq i \\ \text{and } k = n+1, \dots, m.$$

Consistency of conjectures thus implies that

$$\gamma = - |B^{(n+m-1)}| / |A^{(n+m-1)}|$$

From (14):

$$\frac{dX^{n+1}}{dt} \Big|_{t=0} = -n |B^{(n+m-1)}| / |A^{(n+m)}| \\ \frac{dx^1}{dt} \Big|_{t=0} = [ |A^{(n+m-1)}| - (n-1) |B^{(n+m-1)}| ] / |A^{(n+m)}|$$

Substituting these expressions into (15), and rearranging terms, gives

$$\frac{dw}{dt} \Big|_{t=0} = \frac{n(n-1)r_2^1 \lambda [ |A^{(n+m-1)}| - (n+m-1) |B^{(n+m-1)}| ] [ |A^{(n+m-1)}| + |B^{(n+m-1)}| ]}{|A^{(n+m)}| \cdot |A^{(n+m-1)}|}$$

(A5)

Applying the lemma proved above to expression (A5) yields:

$$\left. \frac{dw}{dt} \right|_{t=0} = n(n-1)r_2^1 \lambda(\alpha - \beta)^{n+m-2} / |A^{(n+m-1)}|. \quad (A6)$$

Stability of the market equilibrium requires that the principle minors of  $A^{(n+m)}$  alternate in sign, the first one negative.<sup>9</sup> Therefore,  $\alpha < 0$ , and  $\alpha^2 - \beta^2 > 0$ , which implies  $|\alpha| > |\beta|$ , and hence  $\alpha - \beta < 0$ . Therefore  $(\alpha - \beta)^{(n+m-2)}$  is positive if  $n+m$  is even and is negative otherwise. Similarly,  $A^{(n+m-1)}$  is the  $n+m-1$  principle minor of  $A^{(n+m)}$ , which is positive if  $n+m$  is odd and negative otherwise. Since  $r_2^1 < 0$  and, from the first-order condition (13a),  $\lambda = c^1 / (1 - t) > 0$ , we conclude that the right-hand side of (A6) is positive for  $n > 1$ .

Q.E.D.

Footnotes

1. In Spencer and Brander (1982), the authors study a two-stage game in which a capacity or R&D investment is made at a stage prior to production. In such a setting, export subsidies and R&D subsidies are each welfare improving if implemented separately, but an optimal policy package involves an export subsidy and an R&D tax. Brander and Spencer (1982b) extend the basic argument for intervention to situations in which duopolistic competition takes place in the home market. In such cases an import tariff is often beneficial.
2. Restricting attention to output rivalry entails no loss of generality, however. Kamien and Schwartz (1983) demonstrate that any conjectural variations equilibrium (CVE) in quantities has a corresponding CVE in prices.
3. Analysis of government policy in international markets is typically based on this assumption. See, e.g., Spencer and Brander (1982). It may be justified by specifying the political process of establishing policy as time-consuming and costly, or by endowing the government with a reputation for adhering to announced policy.
4. The second-order condition for a maximum is satisfied locally as long as (i) the home firm's first and second order conditions for profit maximization are satisfied and (ii) the foreign firm's actual response to a change in  $x$  does not differ significantly from the response conjectured by the home firm.
5. We henceforth drop the arguments of the revenue and cost functions and their partial derivatives whenever no confusion is created by doing so. The revenue functions and their partial derivatives are understood to be evaluated at the equilibrium value of

( $x$ ,  $X$ ), while the cost functions and their derivatives are evaluated at  $x$  or  $X$ , whichever is appropriate.

6. The second-order condition for a social optimum is satisfied at the free-trade equilibrium if the product-market equilibrium is stable.
7. In a symmetric, free-trade equilibrium all firms produce the same output, and all revenue and cost function are symmetric, so that, for example,  $r^i = r^1$ ,  $i = 1, \dots, n$ . For notational simplicity and with no loss of generality the following analysis is expressed in terms of the output, revenue, and cost functions of the first home firm and  $n+1$ 'st foreign firm.
8. We assume that this integral is bounded.
9. For a discussion of the stability conditions for conjectural variations models of oligopoly, see Seade (1970).

References

- Brander, J.A. and Spencer, B.J. (1982a), "Export Subsidies and International Market Share Rivalry," mimeographed.
- Brander, J.A. and Spencer, B.J. (1982b), "Tariff Protection and Imperfect Competition," in H. Kierzkowski (ed.), Monopolistic Competition in International Trade (Oxford: Oxford University Press), forthcoming.
- Bresnahan, T.F. (1981), "Duopoly Models with Consistent Conjectures," American Economic Review 71(5), pp.934-945.
- Dixit, A.K. (1984), "International Trade Policy for Oligopolistic Industries," Economic Journal Conference Papers, forthcoming.
- Kamien, M.I. and Schwartz, N.L. (1983), "Conjectural Variations," Canadian Journal of Economics 16(2), pp. 191-211.
- Krugman, P.R. (1983), "Import Protection as Export Promotion: International Competition in the Presence of Oligopoly and Economies of Scale," in H. Kierzkowski (ed.), Monopolistic Competition in International Trade (Oxford: Oxford University Press), forthcoming.
- Perry, M.K. (1982), "Oligopoly and Consistent Conjectural Variations," Bell Journal of Economics 13(1), pp. 197-205.
- Seade, J.K. (1980), "On the Effects of Entry," Econometrica 48(2), pp. 479-489.
- Spencer, B.J. and Brander, J.A. (1982), "International R& D Rivalry and Industrial Strategy" Review of Economic Studies, forthcoming.