ASYMMETRIES OF INFORMATION AND LDC BORROWING WITH SOVEREIGN RISK

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Abstract

Borrowing by the less-developed countries on private international credit markets leads to market outcomes which contrast significantly with those observed in corporate finance and lending between developed nations. This paper relates the unenforceability of bond covenants internationally to asymmetries of information between debtors and creditors about the concurrent indebtedness of borrowers. These asymmetries of information in a model of lending with moral hazard are shown to imply the observed short maturity structure of debt, predominance of bank over bond lending, quantity rationing of credit, and exclusion of the lowest income LDCs from the private international capital market.
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1. Introduction

The dramatic rise in the amount and extent of private lending to the less-developed countries following 1973 has received widespread attention. During the current recession, the Western press has often reported that creditors were unaware of the total amount of lending to individual LDCs, thereby incorrectly assessing the risk of default. This paper presents a theoretical model of international lending with sovereign risk which emphasizes the role of asymmetric information about total debt-service obligations between creditors and debtors. The model is used to explain how sovereign risk leads to important special features observed in LDC borrowing.

The characteristics of borrowing by the LDCs on international capital markets have been investigated and described in several recent studies (see, for example, Eaton and Gersovitz (1981b), Fleming (1981), Hope (1981), IMF (1981 and 1980), O'Brien (1981), and Wellans (1977)). Eaton and Gersovitz (1981b), in particular, argue that the threat of possible repudiation of debt by a sovereign country is responsible for the salient differences between market outcomes for LDC borrowing and the nature of loan contracts observed in lending between developed nations and in domestic corporate finance. Access to long-term loans on the Eurobond market is limited to very few non-OPEC LDCs, and most LDCs which receive loans on the private credit market obtain medium-term commercial credit from the major U.S. and European banks. Therefore, LDC debt is typically of shorter maturity than most developed country corporate debt. The lowest income LDCs almost never gain access to the private loan market and rely upon long-term borrowing from official creditors and international agencies. Private creditors are also reported to

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analyze individual countries' credit-worthiness, that is, their ability, or proclivity, to absorb capital inflows and repay debts. Therefore, the adoption of policies intended to signal credit-worthiness by LDC governments is often observed. Credit on international markets is typically quantity-rationed, with countries having higher rates of saving and investment receiving larger loans at lower rates of interest.

Because lenders are often unable to obtain legal remedies for breach of contract in a debtor's political jurisdiction, mutually advantageous contracts common in domestic corporate bond-finance are unenforceable in the international credit market. In the presence of sovereign risk, lenders must rely upon the threatened denial of future credits and the disruption of a debtor's commodity trade or access to trade-finance to discourage the repudiation of debt. In the case of corporate finance, bond covenants and bankruptcy provisions protect creditors from increases in their exposure to default risk created by subsequent borrowing by their debtors. When these contracts are unenforceable, lenders possess imperfect information about the future total debt service obligations of borrowers.

In the model, production is stochastic and debtors maximize the discounted stream of their expected utility from consumption in each period under the constraint that a default leads to the denial of all future credit, and creditors maximize expected profits. This model is similar to one developed by Eaton and Gersovitz (1981a) to demonstrate quantity-rationing of credit in a deterministic setting (they also discuss the model with stochastic income). This paper extends their work by defining and examining the properties of competitive equilibria in loan contracts under alternative assumptions about the information possessed by creditors. Asymmetries of information about total concurrent indebtedness of borrowers between debtors
and creditors are shown to imply the shortened debt maturity structures, predominance of bank over bond lending, relationship between quantity-rationing of credit and rates of investment, and exclusion of the lowest income LDCs from the private international capital market observed in LDC borrowing. Another paper, by Sachs and Cohen (1982), explores the relationship between default risk, the lack of bond covenants and the special features of LDC borrowing. Their approach and explanations differ greatly from those given in this paper, which concentrate on the role of asymmetries of information between creditors and debtors.

The effects of moral hazard and imperfect information on the characteristics of equilibrium contracts are studied in a model of external borrowing. A sizeable and well-known literature on the consequences of imperfect information for market outcomes already exists. Ideas developed in many papers on moral hazard and insurance markets (for example, A.rott and Stiglitz (1982), Helpman and Laffont (1975), Pauly (1974), and Shavell (1979)) are applied to the problem of LDC borrowing with sovereign risk, and some of the results have parallels in this literature.

The basic model is described in the next section, and the existence of equilibrium for the model under the two alternative assumptions about lenders' information is discussed in section 3. Section 4 considers the efficiency of equilibria under each assumption. Section 5 relates the properties of equilibrium for the model to the stylized facts about private lending to LDCs. The last section concludes the paper with some remarks about debt rescheduling in the model.
2. **The Basic Model**

   This section develops the basic model of international lending with default risk. In contrast to a model of domestic credit markets, lenders face sovereign risk: borrowers can repudiate all their debt. Because non-repayment of loans would be optimal for the borrower in any realized state of nature if no penalty were imposed, lenders are assumed to exclude a defaulter from access to any future loans. This strong incentive device induces repayment in some states of nature and default in others (Stiglitz and Weiss (1980) thoroughly discuss the use of this incentive). Although the rescheduling of debt is not an allowed alternative in the basic model, under certain additional assumptions, rescheduling is a preferred alternative to repudiation for both lenders and borrowers in some states of the world.

   In this model, loans can be interpreted as imports of capital goods which cannot be currently consumed. All loans are assumed to be invested and capital lasts only one period. Output from a given amount of borrowing is a random variable and is obtained one period after investments are made. This assumption is a simplifying one which allows an easily solvable dynamic programming problem and can be relaxed to allow borrowers to choose between consumption and investment of loan principals.

   **Borrower Behavior**

   The following assumptions characterize borrowers:

   A1. Net output in each period is a random variable, $y_t$, which increases with the quantity of borrowing from abroad, $b_{t-1}$. The state of nature is characterized by a random variable, $z_t$, a scalar defined over the unit interval, and the $z_t$'s are independently identically distributed. Net output is a function of $b_{t-1}$ and $z_t$, such that
\[ y_t = y(b_{t-1}, z_t), \frac{\partial y}{\partial b_{t-1}} > 0, \frac{\partial^2 y}{\partial b_{t-1}^2} < 0 \]

and \( \frac{\partial y}{\partial z_t} > 0 \). We also assume that \( y(0, z_t) > 0 \) and that \( \frac{\partial y}{\partial b_{t-1}} \) vanishes as \( b_{t-1} \) approaches infinity.

**A2.** Output is not storable, so that current consumption, \( c_t \), equals current net output minus debt-service payments.

**A3.** The borrower's objective function at time \( v \) is

\[ EV_v = E \left[ \sum_{t=0}^{\infty} g^t U(c_{t+v}) \right] \]

where \( U(c) \) is a continuously twice-differentiable felicity function and \( U' > 0, U'' < 0 \), and \( 0 < \beta < 1 \).

The two following assumptions are adopted restricting the form of loan contracts:

**A4.** Debt matures in one period with debt-service obligations at time \( (t+1) \) given by \( R(b_t) \). Typically, \( R(b_t) = (1+r)b_t \).

**A5.** In each period, the borrower selects an amount of loans, \( b_t \), with debt-service obligations, \( R(b_t) \), from those offered by lenders. If repayments fall below debt-service obligations in any period, further loans will not be forthcoming.

The last assumption may be objected to on the basis that defaulters are not excluded from access to international credit markets for evermore.

However, a moratorium on future lending of finite duration to defaulters or the possibility of rescheduling at a cost to the potential defaulter will have the same qualitative incentive to repay effect. The presumption that any insufficiency to cover debt-service obligations in full leads to declaration of a default can be supported by the proliferation of cross-default clauses in international loan contracts. A cross-default clause allows other lenders to
declare all their loans to a borrower in default after one lender declares a default in an effort to cover its losses out of the borrower's assets in countries in which the lender's lien would be recognized. The exclusion from future participation in the credit market is the only cost to default in the basic model. Additional costs, such as denial of trade credits or retaliatory interference with trade will only enhance the incentive to repay.

Under the above assumptions, optimal debtor behavior for any given amount of borrowing and repayment obligations can be described. The borrower's utility in the state of nature $z_t$ is, under default,

\begin{equation}
V_d(b_{t-1}, z_t) = U(y(b_{t-1}, z_t)) + \sum_{j=1}^{\infty} R^j E[U(y(0, z))]
\end{equation}

In the event of repayment, it is

\begin{equation}
V_R(b_{t-1}, R(b_{t-1}), z_t) = U(y(b_{t-1}, z_t) - R(b_{t-1}))
+ B E \max \{V_R(b_t, R(b_t), z_{t+1}), V_d(b_t, z_{t+1})\}.
\end{equation}

Default is optimal in any realized state of the world, $z_t$, such that $V_d(b_{t-1}, z_t) > V_R(b_{t-1}, R(b_{t-1}), z_t)$.

Because utility is increasing and concave and net output is increasing in $z_t$, if default is optimal for some state $z'_t$, it is optimal for all $z_t$ less than $z'_t$, and conversely for repayment. Therefore, there exists a state, $\bar{z}_t$, such that default occurs for all $z_t$ less than $\bar{z}_t$ and repayment for all $z_t$ greater than $\bar{z}_t$. The probability of repayment is

\begin{equation}
P(b, R(b)) = \frac{1}{\bar{z}} dF(z), \text{ where } F(z) \text{ is the cumulative distribution function for } z_t. \text{ Although } \bar{z}_t \text{ need not be unique, the choice of } P(b, R(b)) \text{ will be unique.}
\end{equation}
This paper is concerned with the properties of equilibrium for this model of international borrowing. Because both debtors and creditors seek to solve their same respective maximization problems every period until default occurs (the lenders' objective functions are defined below), equilibrium will entail a stationary state, in which the debt-service obligation schedule is constant across periods and each borrower obtains the same amount of loans in every period prior to default. This will be true whether or not quantity rationing occurs in equilibrium. Therefore, a characterization of equilibrium requires an understanding of how default behavior varies with a stationary (until default actually takes place) quantity of loans and debt-service obligations.

Because the utility function is concave and the production function is monotone increasing in z, the optimal choice of P(b,R(b)) is given by maximizing expected utility with respect to \( \bar{z} \), under the assumption that the loan contract, (b,R(b)), is available every period prior to default (time subscripts will be suppressed for the remainder of this paper because all quantities will take their stationary state values). The probability of repayment can be found by determining the state, \( \bar{z} \), for which the following expression for expected utility is maximized:

\[
EV(b,R(b),\bar{z}) = \int_0^{\bar{z}} U(y(b,z))dF + \int_{\bar{z}}^{\infty} \frac{1}{\sum_{t=1}^{\infty}} \beta^t \int_0^{\infty} U(y(0,z))dF
\]

\[
+ \int_{\bar{z}}^{\infty} U(y(b,z)-R(b))dF + \beta \int_{\bar{z}}^{\infty} \beta \cdot EV(b,R(b),\bar{z}), \text{ or}
\]

\[
= \frac{1}{\beta} \left\{ \frac{1}{\bar{z}} \int_0^{\bar{z}} U(y(b,z))dF + \frac{1}{\bar{z}} \int_{\bar{z}}^{\infty} U(y(b,z)-R(b))dF + \frac{1}{\beta} \int_{\bar{z}}^{\infty} \beta \cdot EV(y(0,z))dF \right\}
\]

(5) \[
EV(b,R(b),\bar{z}) = \frac{1}{\beta} \left\{ \frac{1}{\bar{z}} \int_0^{\bar{z}} U(y(b,z))dF + \frac{1}{\bar{z}} \int_{\bar{z}}^{\infty} U(y(b,z)-R(b))dF + \frac{1}{\beta} \int_{\bar{z}}^{\infty} \beta \cdot EV(y(0,z))dF \right\}
\]

(1 - \beta |dF)
If $F(z)$ is continuous over the unit interval, then $EV(b, R(b), z)$ is continuous and differentiable in $z$ (also, in $b$ and $r$ for $R(b) = (1+r)b$) and attains a maximum over the unit interval for any loan contract. The first-order condition for an interior maximum of (5) over $z$ is

$$R \left[ \int_{0}^{z} \frac{1}{U(y(b, z))} dF + \int_{z}^{1} \frac{1}{U(y(b, z) - R(b))} dF - \int_{0}^{1} \frac{1}{U(y(0, z))} dF \right] = [1 - RP(b, R(b))] \cdot [U(y(b, z)) - U(Y(b, z) - R(b))].$$

Since $1/(1-\beta P) = \sum_{t=0}^{\infty} (\beta P)^t$, an interpretation of this expression is that the expected discounted value of not defaulting in the state of nature $z$ equals the current loss in utility in state $z$ from repayment of the loan. The second-order condition for a maximum over $z$ is easily checked for continuous $F(z)$ and holds at $z$ satisfying (6). The maximization of expected utility with respect to default behavior is straightforward for discontinuous $F(z)$, and expected utility under optimal default behavior is continuous in the quantity lent and rate of interest charged when $R(b) = (1+r)b$. Furthermore, $P(b, r) = P(b, (1+r)b)$ is continuous in both $b$ and $r$ if $F(z)$ is continuous.

Lender Behavior

The following assumptions characterize lenders:

A6. Lenders are perfect competitors and are risk-neutral, or alternatively, the default risk of individual debtors is uncorrelated with the market risk.

A7. Either lenders borrow deposits at a fixed rate of interest, $p$, repaid with certainty, or they are able to lend on an alternative market at a certain rate of return, $p$. We may assume that $p$ is fixed, or that it is determined competitively where there is an upward-sloping supply of deposits curve. For simplicity, $p$ is taken as fixed.3
Each lender, $i$, seeks to maximize its expected profit, given by:

$$E_{i} = b_{i}(1+r)P(b,r) - b_{i}(1+p),$$
where $b$ is the total borrowing by each debtor and $b_{i}$ is the amount lent by lender $i$.

3. **Equilibrium**

In this section, equilibria are compared for the alternate assumptions about lenders' information. In the first of these, creditors are unable to observe the total amount of loans contracted by any debtor in a given period. They only know the amounts of loans they provide each borrower. The second assumption is that creditors observe the total lending to any borrower each period.\(^4\)

Competitive equilibrium for the model under each assumption is defined as a Nash equilibrium in loan contracts. Equilibrium will occur when no new loan can be offered which achieves non-negative expected profits, contingent upon the lender's information about total indebtedness, and is preferred by a borrower to the contracts offered. In the remainder of this paper, a loan contract will be the pair $(b,r)$, where $b$ is total single period borrowing by a representative debtor (all borrowers are alike throughout this section). The total current outstanding debt of a given borrower may be composed of loans from several lenders, but perfect competition implies that the rate of interest charged by each will be the same.\(^5\)

3.1 **Equilibrium Without Observability of Total Indebtedness**

If each lender is incapable of observing the total borrowing by a debtor in each period, then the loan contracts can only specify the rate of interest and the size of that loan and not the total simultaneous indebtedness of the borrower. Because the probability of repayment depends upon the total debt due at a given date for each borrower, a lender can only know the probability
of repayment when a borrower's excess demand for loans is non-positive at the market rate of interest (we presume throughout that debtors' objective functions are known to creditors). When lenders are unable to observe total borrowing, debtors with different concurrent indebtedness cannot be distinguished. Therefore, if an equilibrium exists, it must occur along the demand curve for loans; that is, the rate of interest alone can be used to ration loans.

The demand curve for loans is derived by maximizing expected utility with respect to both the probability of repayment and the quantity of loans for each period prior to a default, taking as given the rate of interest. If the solution for maximizing with respect to repayment choice results in a probability of default of unity, demand for loans will be infinite. For rates of interest at which expected utility is maximized through this behavior, the demand curve is not defined. These will be high rates of interest, and for the general model, there will exist finite solutions to the maximization problem for interest rates below some limit. The borrower prefers to receive this size loan during every period until a state of nature is realized for which default is optimal at the given rate of interest to receiving a larger loan resulting in a higher optimal probability of defaulting and receipt of no future loans.

The first-order conditions for the loan demand curve are (6) and

\[ \frac{3E V}{\partial b} = 0, \text{ or} \]

\[ \frac{\bar{z}}{\int U'(y(b,z)) \frac{\partial y(b,z)}{\partial b} \, dF} + \frac{1}{\bar{z}} \left( \int U'(y(b,z)-(1+r)b) \left( \frac{\partial y(b,z)}{\partial b} - (1+r) \right) \, dF = 0 \]

(8)
The second-order conditions for a maximum are given in the Appendix, where it is also proved that satisfaction of the second-order conditions implies a downward sloping demand curve. If \( F(z) \) is somewhere discontinuous in the unit interval, then loan demand may not be continuous in the interest rate; although it is continuous for continuous \( F(z) \).

Since default is possible, debtors desire to borrow an amount at a given rate of interest given by equation (8). This implies that the expected marginal utility of the last unit of capital borrowed over states of the world for which repayment occurs is negative:

\[
\frac{1}{Z} \int_U (y(b,z)-(1+r)b) \left( \frac{\partial y(b,z)}{\partial b} - (1+r) \right) dF < 0
\]

Therefore, borrowers demand more capital than they would if default were impossible, and if demand is satisfied, capital will be employed beyond the point of equality of its marginal productivity and marginal cost.

Because debtors choose optimally in which states of nature to repudiate their debts, the expected profits of lenders need not rise with either increasing rates of interest or quantity lent. The probability of repayment always declines with an increase in the rate of interest (holding loan quantity fixed) and, except possibly for small total borrowing, with an increase in indebtedness at a constant interest rate (along the demand curve the partial derivative of the repayment probability with respect to \( b \) is negative). These results are given in the Appendix. Furthermore, the probability of repayment goes to zero for large enough \( b \), so that the following proposition holds:

**PROPOSITION 1:** Given assumptions A1-A5, the set of pairs, \((b,r)\), in the positive orthant such that expected profits are non-negative is bounded from below and on the right. If the marginal productivity of loans (capital) is bounded at zero in all states of nature, then this set is bounded above for all positive \( b \).
The set of loan contracts (pairs (b, r)) for which expected profits are non-negative is depicted in Figures 1 and 2. A demand curve for loans in the rate of interest is also indicated.

The proposition can be illustrated by a simple example. If we let \( U(c) = c \), then maximization of expected utility, equation (5), with respect to \( z \) gives the following first-order conditions for corner solutions:

\[
\begin{align*}
\left\{ \begin{array}{l}
(1+r)b = y(b, z) \\
\bar{z} = 1 \\
\end{array} \right. \quad \text{if } R \begin{cases}
< 1 \\
\geq 1
\end{cases} \\
\mathcal{B} \int (y(b, z) - y(0, z)) dF.
\end{align*}
\]

Therefore, default is certain for all \( b \) for rates of interest greater than \( \bar{r} \), where \( \bar{r} \) is given by:

\[
1 + \bar{r} = \lim_{b \to 0} \frac{1}{b} \beta \int_0^1 (y(b, z) - y(0, z)) dF = \beta \int_0^1 \frac{\partial y(b, z)}{\partial b} \bigg|_{b=0} dF.
\]

Also, for strictly concave production functions, there always exists a finite value of \( b \) for any \( r > -1 \) such that \( \bar{z} \) is unity.

The traditional textbook model of the loan market assumes that any borrower is able to obtain all the loans they desire at a parametric rate of interest and equilibrium results when the market clears at a given rate of
Figure 1
interest. Such an equilibrium can be called an interest rate equilibrium. Because the total concurrent indebtedness of different borrowers cannot be distinguished, an equilibrium without observability must occur on the demand curve. If an equilibrium exists, it must be an interest rate equilibrium, since the interest rate alone can be used to ration loans.

Because the set of loan contracts for which expected profits are non-negative is bounded on the right, an equilibrium without observability need not exist; there need not be any point on the demand curve for which expected profits are at least zero.

Figures 1 and 2 illustrate the existence and nonexistence of equilibria without observability, respectively. Point A of Figure 1 is an equilibrium without observability because no loan contract more preferred by borrowers (at a lower rate of interest) can be offered which provides non-negative profits. In Figure 2, the set of contracts for which expected profits are non-negative lies entirely to the left of the demand curve and at no rate of interest will lenders offer a market-clearing quantity of loans. Therefore, no equilibrium exists.

The set of equilibria without observability and the set of interest rate equilibria are identical. For this to hold, a special case of interest rate equilibrium must be ruled out. If the demand curve of Figure 2 intersected the vertical axis, then an interest rate equilibrium would result at that point, since both supply and demand at that rate of interest would equal zero. Such a point would not be an equilibrium without observability because there are loans which can be offered at lower rates of interest that achieve positive expected profits (the point is not a Nash equilibrium in loan contracts). This type of interest rate equilibrium will not occur, because if the demand curve intersected the vertical axis, any borrower would increase their utility by demanding any quantity of loans and defaulting with certainty at that rate of interest. Loan demand
would not be defined at such a rate of interest, so that the demand
curve will never intersect the vertical axis.

The above arguments can be summarized as:

**Proposition 2:** Without observability by lenders of total concurrent
indebtedness of borrowers, equilibrium may fail to exist. If it does exist, equili-
brum without observability is equivalent to the interest rate equilibrium.

The existence or non-existence of an equilibrium provides no information
about the dynamics of contracting; nothing can be inferred about disequilibrium
trading. Even if an equilibrium without observability fails to exist, loan
contracts which achieve positive expected profits generally exist (as in Figure 2).
If no other lender offers credit, any particular creditor can provide a loan
contract with positive expected profits, so that a no lending equilibrium does
not exist, either. Non-existence of an equilibrium does not imply that dis-
equilibrium lending will not occur.

3.2 Equilibrium with Observability of Total Indebtedness

If lenders are able to observe the total borrowing by debtors at each
date, then loan contracts specifying the rate of interest to be paid as a
function of total current borrowing are possible. Competitive lenders will
only offer contracts which achieve non-negative expected profits. A
competitive (Nash) equilibrium will occur when no further contract can be
offered to borrowers which achieves non-negative expected profits and is
chosen by the borrower over another contract already offered.

The existence of a competitive equilibrium with observability of total
concurrent borrowing can be proven for the model. Preferences over loan
contracts are given by maximizing expected utility over the probability of
repayment and ranking contracts using the expected utility of contracts with
the optimal choice of default behavior. This resulting function (an envelope) is continuous for continuous felicity and production functions, although concavity of these functions does not assure concavity of expected utility with optimal default behavior (for example, it generally will not be if \( F(z) \) is not everywhere continuous in \([0,1]\)).

**THEOREM:** An equilibrium with observability exists for the model if \( F(z) \) is continuous on \([0,1]\). This equilibrium is equivalent to an equilibrium in exclusive loan contracts (i.e., each borrower can select only one contract each period and lenders are competitive).

**PROOF:** Let \( X \) be the set of loan contracts, \((b, r)\), in the positive orthant which achieve non-negative expected profits. \( X \) is bounded from below and on the right. If \( F(z) \) is continuous everywhere on \([0,1]\), then \( P(b, r) \) is continuous in \((b, r)\), so expected profits, \( E \pi \), are continuous over the positive orthant. Therefore, \( X \) is closed. For any quantity of loans, \( b \), borrowers prefer contracts at lower rates of interest, so that expected utility will be greatest over the lower and righthand boundaries of \( X \). Therefore, a maximum of \( EV \) over \( X \) exists.

An equilibrium with observability is depicted in Figure 3. The curve denoted, \( EV_0 \), is an indifference curve over loan contracts under optimal default behavior. Expected utility increases in a downward and outward direction, and each indifference curve will be tangent to a horizontal line at the demand curve because expected utility is maximized there for each rate of interest if there is no constraint on the availability of credit. Equilibrium occurs at point A in Figure 3.

An equilibrium with observability will typically entail credit rationing. Equilibrium contracts occur inside the demand curve, except in a special case
(when the locus of loan contracts providing zero expected profits has zero slope where it crosses the demand curve). In this instance, equilibria with and without observability coincide.

The condition that \( F(z) \) be continuous over the unit interval assured that the set of loan contracts for which expected profits were non-negative was closed. If \( F(z) \) is discontinuous (for example, if only a discrete set of states of the world occurred with positive probability), then this set need not be closed. Because there can be contracts for which two different probabilities of repayment yield equal reward to the borrower (the debtor can be indifferent between repaying and defaulting in a particular one of a discrete set of states occurring with positive probability), expected profits are not well-defined at some contracts. Even though expected utility with optimal default behavior is continuous, so that indifference curves are continuous, they are not necessarily concave and the demand curve is generally discontinuous. Therefore, neither type of equilibrium may exist without continuity of \( F(z) \).\(^6\)

3.3 Efficiency of Equilibrium

Equilibria with and without observability are depicted in Figure 4. The equilibrium without observability is at Point A, and the one with observability, at B. The curve \( EV_0 \) and \( EV_1 \) are indifference curves over loan contracts after maximization with respect to default-repayment choice. Because an equilibrium with observability is a contract which maximizes expected utility over the subset of contracts for which expected profits are non-negative, it cannot be Pareto-dominated by an equilibrium without observability, if one exists, since this latter contract must be in the same subset of the plane. If the supply curve of deposits is upward-sloping, then
Figure 4
the market-clearing certain rate of interest may be lower with observability. The set of loan contracts which provide non-negative expected profits will enlarge, and borrowers' expected utility will further increase.

Under the assumptions of the model, an equilibrium with observability is a constrained-optimum for both lenders and borrowers. A constrained-optimum is defined as an allocation such that no agent's welfare can be improved without decreasing that of another given the allocations achievable with the contracts and information available. That is, debtors make their optimal choice between repayment and default under the equilibrium loan contract in each state of nature. This choice is made *ex post*, after the state is revealed.

**PROPOSITION 3**: Under the assumptions of the model, an equilibrium with observability of total concurrent indebtedness is a constrained-optimum.

If other markets are considered, this result may no longer hold. The move from one type of equilibrium to the other may induce changes in the international prices of commodities, leading to net welfare reductions for some debtor countries.

The failure of efficiency of equilibrium without observability arises because lenders are unable to restrict the quantity of loans provided each borrower at a given rate of interest. An individual lender does not internalize the full effect on the probability of repayment of debt of an increase in the amount he lends to a given debtor. At a given rate of interest, the size loan which maximizes the first creditor's profits will generally be less than the total amount of loans which achieve non-negative expected profits. This precunary externality leads to the failure of efficiency.
4. Implications for LDC Borrowing in Private Credit Markets

Properties of equilibria for the model under the asymmetries of information which result from the absence of bond covenants establishing bankruptcy provisions and restricting debt dilution imply many of the differences between LDC and domestic corporate borrowing. The shortening of the maturity structure of debt, predominance of bank over bond lending, quantity rationing of credit, and redlining of the poorest LDCs are considered below.

4.1 Shortening of Debt Maturities

With longer-lived capital and multiple debt maturities, the results of the previous sections imply that the lack of enforceable constraints on debt dilution in international lending leads to an equilibrium with debt of short maturities. Between the negotiation and maturation of long-term debt, information about the ultimate repayment ability of a borrower is usually revealed. Providers of shorter term loans which mature concurrently with long-term debt are more informed about the profitability of lending to particular debtors. Without enforceable contracts restricting subsequent borrowing and establishing debt priorities in the event of bankruptcy, these asymmetries of information can lead to the observed short maturity structure of LDC borrowing.

Suppose that a loan which matures in two periods and provides zero expected profits is made and that after one period, it is known whether realized output will be in a lower or an upper portion of its range. In the former case, the expected profits on the two-period loan and on any one-period loan which matures at the same date are negative after the information is revealed. In the latter case, there are one-period loan contracts which achieve non-negative expected profits. When subsequent equilibrium one-period
loans are correctly anticipated, the original two-period loan provides
negative expected profits. Therefore, the set of longer term loan contracts
which achieve non-negative expected profits is a subset of what it would be
in the presence of bond covenants. Because the set of loan contracts which
provide non-negative expected profits is bounded from above in the rate of
interest, the rate of interest on longer term debt cannot necessarily be
raised in order to attain positive expected profits. For this reason, the set
of long-term loan contracts achieving non-negative expected profits may be
empty. With increasing debt maturities, this possibility becomes more
likely.

Another possible reason long maturity debt is not observed is indicated
by the above observations. As noted by Sachs and Cohen (1982), a long-term
loan contract would require a higher rate of interest for a given size loan
than a shorter term contract to achieve non-negative profits. Borrowers may
prefer debt of shorter maturities at lower rates of interest, so that
long-term loan contracts do not occur in a competitive equilibrium. However,
their model presumes that the interest rate can always be increased to achieve
non-negative expected profits.

A natural way to endogenize debt maturities is to use a neoclassical
growth model with stochastic technology and optimal saving (with sovereign
risk, this type of model is analytically troublesome; see footnote 1).
Because the probability of future repayment of debt will depend upon the
current capital endowment, providers of short maturity debt will possess
better information about this probability than lenders of longer maturity debt
(due concurrently). Unfortunately, with multiple maturities of debt, the
stochastic dynamic programming problem will no longer be Markovian. The terms
of any loan contract will depend upon the entire history of capital
accumulation back to the issue of the oldest unsettled debt, so that
determination of a stochastic equilibrium would be extremely combersome,
if possible.

An alternative way to demonstrate the effects of sovereign risk
on debt maturities is to allow capital to last two periods in the basic
model and consider debt which matures after either one or two periods.
If partial information is available in any period about what state of
nature will prevail in the next, then lenders of debt which matures
in one-period will be better informed about the expected profits of
various loan contracts than will lenders of debt maturing in two periods.
This extension of the basic model is presented using an example in the
Appendix.

4.2 Predominance of Bank Lending

In the absence of enforceable contracts limiting simultaneous debt of
equal priority or establishing compensation procedures in the event of default,
a bond market in LDC debt is unable to provide lenders with observability
of the total concurrent indebtedness of individual borrowers. The analysis
of the existence and efficiency properties of equilibria with and
without observability demonstrates the benefits to lenders of observing
total debt-service obligations. If the market achieves an equilibrium
without observability and some creditor is able to restrict the total
current borrowing of a particular debtor, then there exists a loan
contract which provides both positive expected profits to the lender
and a higher expected utility for the debtor than attained with that
equilibrium contract. Therefore, whether or not a bond market equili-
brum exists, creditors have an incentive to achieve observability of
total concurrent indebtedness. While lending through bonds does
not accommodate observation of each debtor's total borrowing, bank lending with disclosure of amounts can allow creditors to obtain this information.

Lenders' efforts to acquire information about the characteristics of individual borrowers have been widely recognized; however, in the model, each lender knows the utility and production functions of every debtor (the relationship between the probability of repayment and all current loan contracts for each borrower is known). Of special importance for LDC borrowing on private credit markets is information about total borrowing. Bank lending is predominant because, unlike bondholders, intermediaries have the ability to restrict simultaneous debt-service obligations by informing each other of the terms of their individual loans to any given debtor. The model demonstrates that each lender has an incentive to reveal this information to every other lender.

In addition to the predominance of bank lending, these comparisons also explain the occurrence of consortium lending to the LDCs. If each borrower is able to obtain a loan from only one source (a consortium) each period, a competitive equilibrium in such loan contracts will be achieved with each borrower's most preferred contract within the set of ones which provide non-negative expected profits. This is exactly the same set of equilibria (since equilibrium loans are possibly not unique) as that achieved with observability of total concurrent indebtedness.

In the model, borrowers repay loans in some states of nature because they are denied all future access to credit in the event of a default. The enforcement of this incentive mechanism, as similar ones which allow debt-rescheduling and further credit, requires cooperation between potential lenders. Such cooperation is easier to attain the smaller the number of creditors and may be impossible to achieve with bond-finance. Because the possible denial of short-term trade-finance will increase the cost of
defaulting to a borrower, banks which provide such finance will face lower probabilities of default for the same loan contracts as will other lenders. Therefore, the predominance of bank over bond lending may result because of the ability of intermediaries to cooperate and impose stronger repayment incentives, as well as to exchange information at lower cost.

4.3 Quantity-rationing of Credit

The solution of the model with observability of concurrent indebtedness demonstrates how the threat of debt repudiation leads to the quantity-rationing of credit. In the case of domestic corporate finance, enforceable bond convenants and bankruptcy provisions which provide this information to creditors allow a market in corporate bonds to exist, so that, in general, the interest rate alone rations credit. The exchange of information between financial intermediaries and short maturity structures which increase observability in the presence of default risk have the opposite effect in LDC borrowing, leading to quantity-rationing.

In the above sections, stationary state equilibria are described for the model under the assumption that all borrowed funds are invested. The model can be rewritten to allow debtors to choose between the consumption and investment of loans they receive. Allowing consumption out of current borrowing reduces the probability of repayment for each loan contract. Debtors with lower rates of discount can be shown to optimally select higher rates of investment resulting in an outward shift in the locus of loan contracts for which expected profits vanish and an inward shift in their demand curves for loans. Therefore, in an equilibrium with observability, these borrowers receive larger loans with probably lower rates of interest and
are rationed less severely than borrowers with lower propensities to invest. This result coincides with the observation that international differences in savings-investment rates and available investment opportunities lead to significant variations in the amount of lending to different countries, along with variations in risk premia charged.

4.4 Exclusion of Lowest-income LDCs from the Private Capital Market

The model can also explain the exclusion of the lowest-income countries from access to loans on private international credit markets. If investment possibilities (that is, the production function, \( y(b,z) \)) vary across countries, the highest expected rate of return on any loan to some countries can be lower than the expected rates of return on many loans to other countries. Then, for an upward-sloping supply of deposits schedule, the expected rate of return for which the loan market is in equilibrium and the quantity lent equals the supply of deposits forthcoming at that rate can be greater than the maximum attainable rate of return on any loans to some countries (this explanation is similar to one given by Stiglitz and Weiss (1981)).

5. Conclusion

Although actual defaults may occur rarely, the threat of debt repudiations significantly affect market outcomes for LDC borrowing. The properties of equilibria with and without observability of the total concurrent indebtedness of borrowers by lenders imply many of the observed differences between international lending and domestic corporate finance. In particular, asymmetries of information in the model of lending with moral hazard are shown to imply the lack of bond lending and short maturities of debt in this market.
The rescheduling of LDC debt has not been addressed in this paper. Instead, repayment on schedule and default are the only options available to debtors. A proper inclusion of rescheduling in this type of model would be to derive the optimal strategy for lenders endogenously. While debt rescheduling is superior to default for lenders and could be combined with the availability of future credit to induce debtors not to default, the possibility of rescheduling will also reduce incentives to invest and repay debts on schedule. Particular forms of rescheduling have been introduced into the model which expand the set of contracts which creditors can offer (a contract then includes a specification of rescheduling opportunities). Because the original full set of contracts providing non-negative expected profits can still be offered (these involve no rescheduling), a Nash equilibrium contract in the larger set will be preferred by debtors and achieve non-negative expected profits.

In the model, debts are repaid with positive probability because every creditor refuses to lend to a past defaulter. A moratorium on future lending to defaulters is not achievable as a Nash equilibrium. If every other creditor observes such a moratorium for some number of periods, then any particular lender can offer a profitable loan contract to a recent defaulter since the incentive to repay is still enforced by other creditors. A Nash equilibrium fails to exist even at a moratorium length of zero: when no other lending occurs, any creditor will want to be the sole lender because they can enforce their own moratoria.

The debtor behavior described in section 2 can be modified trivially for the case of finite length moratoria. Although an increase in the length of the moratoria reduces the probability of default, by increasing the time that any potential borrower in default is excluded from the market, the net effect can be a reduction in the average number of actual borrowers present at any time. A creditor optimal cooperative equilibrium in debt moratorium length with perfect competition in loan contracts can be determined by maximizing the expectation of profits with respect to this length.
Appendix

Comparative Statics of Debtor Behavior

The first-order conditions for a maximum of expected utility at a given rate of interest for continuous \( F(z) \) are

\[
\frac{\partial EV}{\partial z} = 0, \quad \text{or} \quad (1-\beta P) \cdot \Delta U(b, r, z) = \beta[A(b, r, z) - EU_0]
\]

\[
\frac{\partial EV}{\partial b} = 0, \quad \text{or} \quad \frac{\partial A(b, r, z)}{\partial b} = 0
\]

where \( P = \frac{1}{z} \int dF(z) \),

\[
\Delta U(b, r, z) = U(y(b, z)) - U(y(b, \tilde{z}) - (1+r)b)
\]

\[
A(b, r, z) = \int_0^z U(y(b, z))dF + \int_{\tilde{z}}^1 \frac{1}{z} U(y(b, z) - (1+r)b)dF,
\]

\[
EU_0 = \int_0^1 U(y(0, z))dF.
\]

The second-order conditions evaluated at a point satisfying the first-order conditions are:

\[
\frac{\partial^2 EV}{\partial z^2} = (1-\beta P) - 1 \frac{\partial^2 U}{\partial z^2} f(z) < 0,
\]

\[
\frac{\partial^2 EV}{\partial b^2} = (1-\beta P) - 1 \frac{\partial^2 A}{\partial b^2} < 0, \quad \text{and}
\]

-
\[ \frac{a^2EV}{ab^2} - \frac{a^2EV}{az^2} - \left( \frac{a^2EV}{abz} \right)^2 = (1-\beta p)^{-2} \left[ \frac{a^2L}{ab^2} \frac{aU}{az} f(\tilde{z}) - \left( \frac{aU}{ab} f(z) \right)^2 \right] \]

> 0, where \( f(\tilde{z}) = \frac{dF}{dz} \) evaluated at \( \tilde{z} \).

For concave \( U(c) \) and \( y(b, \tilde{z}) \) in \( b \), (22) and (23) are satisfied because

\[ \frac{a\Delta U}{a\tilde{z}} = [U'(y(b, \tilde{z})) - U'(y(b, \tilde{z}) - (1+r)b)] \frac{ab}{a\tilde{z}} f(\tilde{z}) < 0, \]

\[ \frac{a^2A}{ab^2} = \int_0^{\tilde{z}} \left( \frac{\Delta y}{ab^2} \right)^2 dF + \int_{\tilde{z}}^1 \left( U'(y(b, z)) - (1+r)b \frac{\Delta y}{ab^2} \right) dF \]

\[ + \int_0^{\tilde{z}} \left( \frac{\Delta y}{ab} \right)^2 dF + \int_{\tilde{z}}^1 \left( U''(y(b, z)) - (1+r)b \left( \frac{\Delta y}{ab} - (1+r) \right)^2 dF \right. \]

< 0.

Equation (24) is equivalent to

\[ \frac{a^2A. a^2A}{ab^2. az^2} - \frac{a^2A}{abd\tilde{z}} > 0. \]

Therefore, satisfaction of the second-order conditions is equivalent to concavity of \( A(b, r, \tilde{z}) \) at a point where (20) and (21) hold.

If \( F(z) \) is not continuous, maxima of EV still exist over \( \tilde{z} \) and \( b \). For example, if only finite \( z \) occur with positive probability, expected utility is given by:

\[ EV = \sum_i f(z_i) V_r(b, r, z_i) + \sum_j f(z_j) V_d(b, z_j), \]
where \( f(z_i) \) = probability of state \( i \), and \( i \) ranges over states of repayment and \( j \) ranges over states of default.

The first-order condition for maximization with respect to \( b \) is

\[
(28) \quad \sum_{i} f(z_i) \left[U'(y(b,z_i)) - (1+r)b \left( \frac{\partial y}{\partial b} - (1+r) \right) \right] \\
+ \sum_{j} f(z_j) \left[U'(y(b,z_j)) \frac{\partial y}{\partial b} \right] = 0.
\]

The second-order condition is satisfied for concave \( U(c) \) and \( y(b,z_i) \) in \( b \).

The slope of the demand curve is given by total differentiation of (20) and (21) with respect to \( r \), which yields

\[
(29) \quad \frac{db}{dr} = \frac{\left( \frac{\partial^2 A}{\partial b \partial z} \right) \left( \frac{\partial A}{\partial r} - (1-\beta) \frac{\partial U}{\partial r} \right) - (1-\beta) \frac{\partial U}{\partial z} \frac{\partial^2 A}{\partial b \partial z} \left( \frac{\partial A}{\partial z} \frac{\partial^2 A}{\partial b \partial z} \frac{\partial A}{\partial z} \frac{\partial A}{\partial b} \right)}{(1-\beta) \left( \frac{\partial U}{\partial z} \frac{\partial^2 A}{\partial b \partial z} - \frac{\partial^2 A}{\partial b \partial z} \frac{\partial U}{\partial z} \frac{\partial A}{\partial b} \right)}
\]

\[
(30) \quad \frac{dz}{dr} = \frac{\partial^2 A}{\partial b \partial z} \left( \frac{\partial A}{\partial r} - (1-\beta) \frac{\partial U}{\partial r} \right) + (1-\beta) \frac{\partial U}{\partial b} \frac{\partial^2 A}{\partial b \partial z} \left( \frac{\partial A}{\partial z} \frac{\partial^2 A}{\partial b \partial z} \frac{\partial A}{\partial z} \frac{\partial A}{\partial b} \right) \left( \frac{\partial U}{\partial z} \frac{\partial^2 A}{\partial b \partial z} - \frac{\partial^2 A}{\partial b \partial z} \frac{\partial U}{\partial z} \frac{\partial A}{\partial b} \right)
\]

where

\[
(31) \quad \frac{\partial A}{\partial r} = -b \int \frac{1}{z} U'(y(b,z)) - (1+r)b \, dF < 0,
\]

\[
(32) \quad \frac{\partial U}{\partial r} = bU'(y(b,z)) - (1+r)b > 0.
\]
\[
\frac{a^2_A}{\partial b \partial z} = f(\bar{z}) \left( U'(y(b, \bar{z}) \frac{\partial y}{\partial b} - U'(y(b, \bar{z}) - (1+r)b) \left( \frac{\partial y}{\partial b} - (1+r) \right) \right),
\]

\[
\frac{a^2_A}{\partial b \partial r} = \frac{1}{\bar{z}} \int U'(y(b, \bar{z}) - (1+r)b + bU''(y(b, \bar{z}) - (1+r)b) \left( \frac{\partial y}{\partial b} - (1+r) \right) dF.
\]

The denominator in (29) and (30) is positive if the second-order conditions for a maximum are fulfilled. Additionally, along the demand curve

(33) \[ \frac{a^2_A}{\partial b \partial z} > 0. \]

Therefore, using (31), (32), (33), if (30) is either positive or negative, (29) must be negative. The probability of default either rises or falls with increasing \( r \) along the demand curve.

The effects of changes in \( b \) and \( r \) on the probability of repayment are found by differentiation of (20). These are

(34) \[ \frac{\partial \bar{z}}{\partial r} = \frac{b \frac{\partial A}{\partial r} - (1-BP) \frac{\partial \Delta U}{\partial r}}{(1-BP) \frac{\partial \Delta U}{\partial \bar{z}}} > 0, \]

so that \( \frac{\partial p}{\partial r} < 0 \), and

(35) \[ \frac{\partial \bar{z}}{\partial b} = \frac{b \frac{\partial A}{\partial b} - (1-BP) \frac{\partial \Delta U}{\partial b}}{(1-BP) \frac{\partial \Delta U}{\partial \bar{z}}}. \]

For small \( b \), (35) can be negative, but for \( b \) such that \( \frac{\partial A}{\partial b} = 0 \), \( f(\bar{z}) \frac{\partial \Delta U}{\partial \bar{z}} = \frac{a^2_A}{\partial b} \) will be positive if \( y(b, \bar{z}) \) is continuous, so that (35) is positive and \( \frac{\partial p}{\partial b} < 0 \).
The probability of repayment is defined by

\[ \Delta U(b, r, \bar{z})(1-\beta P) = \beta(A(b, r, \bar{z}) - EU_0) \]

for an interior solution. Because the \( \lim_{b \to 0} \frac{\partial y(b, z)}{\partial b} = 0 \) and \( \frac{\partial y(b, z)}{\partial b} \) is bounded at \( b=0 \), the quantity \( (y(b, z)-(1+r)) \) becomes negative for large \( b \) and for large \( r \), for all values of \( z \). Since the righthand side of (2) is at most \( \frac{1}{\beta} \int (U(y(b,z)) - U(y(0,z))) dF \), for all \( r > -1 \), there exists a value for \( b \) such that \( \bar{z} = 1 \) and for all positive \( b \), there exists a value for \( r \) such that \( \bar{z} = 1 \) and \( P = 0 \). Therefore, expected profits are negative for large enough total concurrent lending and for large enough rates of interest, and the set of loan contracts for which \( E_\pi > 0 \) is bounded from the right and from above.

**Extension to Allow Multiple Debt Maturities**

A way to demonstrate the effect of sovereign risk on debt maturities is to allow capital to last two periods in the basic model and consider debt which matures after either one or two periods. Suppose that in any period information is revealed indicating whether \( z \) for the next period will be in the upper or lower half of the unit interval; for more than one period ahead, the value of \( z \) is known to lie in the upper or lower half with probabilities \( P \) and \( (1-P) \), respectively. As in the example of section 3, let \( U(c) = c \), and let output be given by

\[ y_t = y(b_1, b_2, z_t) \]

where \( b_1 \) is the total amount lent at \( t-1 \), and \( b_2 \) is the total amount lent at \( t-2 \).

The probability of repayment in any period is given implicitly by

\[ R = y(b_1, b_2, \bar{z}) \]

where \( R \) is the total of current debt-service obligations.
A condition analogous to (9) determines pairs of loan contracts for which default is certain.

Equilibrium one-period maturity loan contracts depend upon the total amount lent the period before (regardless of maturation), the terms of two-period debt due at the same date, and the information revealed about the value of z at maturity. Given the terms of loans made the previous period, the expected profits for the two one-period contracts are:

\[(17) \quad \Pi_1 = (1+r_1)b_1 \frac{1}{z} \int dF - (1+p)b_1 , \text{ and} \]

\[(18) \quad \Pi_1' = (1+r_1')b_1' \frac{1}{z'} \int dF' - (1+p)b_1' , \text{ where } z \text{ and } z' \text{ are given by (16).} \]

Expected profits for a two-period loan \((b_2, r_2)\) are:

\[(19) \quad \Pi_2 = (1+r_2)^2b_2 \left( P \cdot \frac{1}{z} \int dF + (1-P) \frac{1}{z'} \int dF' \right) - (1+p)^2b_2. \]

For large enough repayment obligations for the two-period loan, \(z'\) will equal 1/2 and \(b_2'\) will be zero.

Since increasing the rate of interest reduces the probability of repayment (and eventually assures default), expected profits for two-period debt may always be negative for positive loan amounts even though one-period loan contracts achieve non-negative expected profits.

Because the probability of repayment of two-period loans is a fraction of that for subsequent one-period debt, there exist concave production functions such that condition (16) is satisfied for either one-period loan contracts, these contracts achieve zero expected profits, and at the interest rate maximizing two-period expected profits, these profits are negative.
Footnotes

1. The assumption that capital lasts only one period is a greatly simplifying one. A model with capital accumulation and optimal savings with sovereign risk has been studied. In that model, capital accumulation is stochastically optimal with foreign borrowing until a default occurs (an expected utility maximizing decision); during a lending moratorium, savings is chosen optimally without borrowing. The addition of capital accumulation with sovereign risk does not alter the conclusions of the present paper and greatly complicates the analysis.

2. Alternatively, \( P(b, R) \) can be found by equating \( V^d \) and \( V^r \) and for \( z \). The assumptions on the utility and production functions imply that \( V^d(b, z) \) is decreasing in \( z \) and \( V^r(b, R, z) \) is increasing in \( z \). Therefore, \( z \) is just the state (possibly non-unique) for which \( V^d \) and \( V^r \) are equal.

3. The determinants of the supply of funds from the developed (and oil-exporting) countries is outside the scope of this paper. Although the quantity lent to the LDCs as a whole may influence the deposit rate of interest, other factors, especially the economic performance of the developed countries, are primary determinants of this rate. The vast majority of funds lent by private intermediaries to the LDCs are received as interbank deposits. It is reasonable to assume an exogenous supply of deposits in the expected rate of return and that all lenders pay the same deposit rate.

4. These distinctions between competitive equilibria with and without observability are made in the important study of moral hazard in insurance markets by Arnott and Stiglitz (1982).
5. The assumption that moratoria on future lending to defaulters last forever leads to minor algebraic simplification. The effects of imperfect information about concurrent debt-service obligations on market outcomes appear in individual loan contracts. In addition to the determination of equilibrium loan contracts in the text, a description of market equilibrium (which will be stochastic) requires a knowledge of the distribution of the number of actual borrowers (those not under moratoria) at any particular time. If the debt moratorium is of finite duration, then a limiting distribution can be shown to exist if either the supply of deposits is perfectly elastic or upward-sloping. In the former case, explicit calculation of the distribution at any time is possible and the solution to the debtor's maximization problem given in section 2 only needs to be altered for the finite moratorium length (this is trivial). In the latter case, the limiting distribution cannot be given, in general, and the debtor's maximization problem must take into account a distribution over the terms of future loan contracts (because the deposit rate fluctuates).

6. The stochastic model in Eaton and Gersovitz (1981a) does not have a competitive equilibrium for this reason. In their example, the set of loan contracts which achieve non-negative expected profits is open.
References


