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THE SUBSTITUTION BETWEEN MALE AND FEMALE LABOR
IN RURAL INDIAN AGRICULTURAL PRODUCTION

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ABSTRACT

Flexible functional form of production functions for each of three crops grown in the ICRISAT Indian village sample were estimated using fixed effects methods. Production functions, rather than the derived first-order conditions (input demand functions) or the dual cost functions, accommodated the panel nature of the data, as well as the limitation of lack of variation in prices. Production was simply modeled as a two-stage process that corresponded to the observations of agriculturalists, to wit, the harvesting stage was a Leontief-type process. The marginal product of female labor -- both family and hired -- was found to be significant in the production of three major crops grown in the six sample villages. Female and male labor were found to be asymmetrically substitutable with respect to all other factors of production, indicating that the (conventional) aggregated labor input is statistically inappropriate. Certainly, the disaggregated specification enhances our understanding of the productive relationship between factors.

The Substitution Between Male and Female Labor
in Rural Indian Agricultural Production

Flexible functional forms of an agricultural production function are estimated in order to determine the nature of the production relationships between the labor of men and women and other factors. The production functions are unconventional. Varying farmer efficiency and agricultural technologies, along with the division of labor (between men and women) are incorporated in the specification. Panel data from six villages in south central rural India accommodate the unconventional model. The titular concern is the validity of aggregating the labor inputs of men and women. The most obvious limitation in the application of the results when estimating such production functions is the imposed symmetry of the substitution elasticities between factors. Nor can the marginal product of the labor of women be distinguished from that of men. At the very least, disaggregation of the labor of men and women would allow assessment of the importance of the contribution of women to agricultural production.

The organization of the paper is conventional. The empirical model is presented in Part I, and the data analyzed are described in Part II. The results, conclusions, and summary follow in Parts III, IV, and V.

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I. Empirical Model

Physical production functions, rather than dual cost or profit functions, are estimated. The production functions are embedded in the model. The utility of the farm household is maximized subject to, among others, the production constraints. The wages of the family members who work on the family land thus are endogenous. Only the wages of the hired farm laborers are observed. Further, there is little variation in prices in the data. Farm households from six small villages in south central India were surveyed over three years. So, physical production functions were estimated in order to calculate the marginal productivities of the input factors along with the substitution elasticities between factors. Output is expressed in kilograms of crop, labor inputs in hours worked, and land in acres per plot sown.

The production functions were modeled to reflect certain real world phenomena: (1) the techniques used and (2) the division of labor in agricultural production, and (3) the variation in farmer efficiency.

(1) Techniques

Agriculturalists at ICRISAT likened the harvesting stage technology to fixed proportions. The farmer sizes up his standing crop, and contracts for sufficient labor to harvest the crop. The production technology modeled was thus simplified to two stages. The first stage combines ploughing, tilling, sowing, weeding and thinning, interculturing, plant protection and watching, to produce the standing crop. The standing crop is combined with harvest, threshing, and processing inputs to produce the final product.

In the first stage, the standing crop is produced according to production

function f , with vector of inputs v .

$$y_1 = f(v_1) \quad (1)$$

In the second stage, harvest output y_2 is produced according to some other production function, g , combining standing crop y_1 with harvest inputs v_2 ,

$$y_2 = g(f(v_1), v_2) \quad (2)$$

As long as the two stages are separable, the production parameter can be estimated (up to a scale), by regression of harvested output y_2 on the vector of first stage inputs, v_1 ,

$$y_2 = g(f(v_1)) \quad (3)$$

because dg/df is constant. A fixed proportion technology would be sufficient for separability. The assumption of separability of stages makes estimation tractable, but it can be tested just as separability of male and female labor.

(2) Division of Labor

Along with the production technology observed by the scientists, the division of labor between men and women was modeled. Labor inputs of men and women in all the tasks comprising the first stage, along with animal power, machines, and land plot size, were distinguished.

Flexible functional forms of the production function were estimated rather than the conventional constant elasticity of substitution (CES) or the Cobb-Douglas (CD). Both the CES and the CD restrict the substitution possibilities between factors to be constant, and equal to one,

respectively. Further, if the labor of men and women were equally substitutable with other inputs, that is, if the marginal rate of substitution (MRS) between the labor of men and women were independent of the level of other inputs, then a consistent aggregate labor input exists. Alternatively, if the labor of men were, say, more complementary with animal power than the labor of women -- men drive the ploughs drawn by bulls -- then the marginal rate of substitution between male and female labor would not be independent, and male and female labor could not be consistently aggregated. Consistency is an assumption implicit in the CES and CD specification that can be tested when data are available that disaggregate male and female labor inputs. (Taylor (1982) showed that of the separability of male and female labor are equivalent to tests of the independence of the MRS of male and female labor¹ to the level of all other inputs and tests of the equality of the substitution elasticities, and are implemented by imposing non-linear restrictions on the parameters of the production function.²)

In general, the true production function

$$y = F(v_i) \tag{4}$$

can be approximated at a point by a second order Taylor Series expansion around a certain point. When the point is zero, the flexible form is the generalized quadratic,

$$y = \beta_0 + \sum_i \beta_i v_i + \sum_i \sum_j \beta_{ij} v_i v_j \tag{5}$$

The form should be well behaved (displaying monotonicity and convexity) around the sample mean. The relevant virtue is that the separability of

inputs in the labor partition -- male and female -- with respect to all other inputs is not imposed, so separability may be tested. When testing (weak) separability of labor inputs with respect to all other non-labor inputs, the parameter restrictions are simple.

Let v_1 represent male labor hours, v_2 , female labor hours, v_3 , animal power, and v_4 , land. To test weak separability of the form

$$y = F(h(v_1, v_2), v_3, v_4)$$

two independent restrictions are imposed,³

$$\beta_1 = \beta_{13} \beta_2 / \beta_{23} \quad \text{and}$$

$$\beta_{14} = \beta_{13} \beta_{24} / \beta_{23}$$

$$\beta_i \text{ and } \beta_{ij}, \quad i = 1 \dots 4, \text{ and } j = 1 \dots 4.$$

are the coefficients of the generalized quadratic (Equation 5).

Valid imposition of the restrictions is equivalent to pairwise equality of the Allen-Uzawa (AU) substitution elasticities between labor and non-labor inputs. Recall that the AU elasticity is a measure of the technical relationship between two factors, measuring the extent to which factor proportions change in response to changes in their marginal rate of substitution. When competition and profit maximization are assumed, the elasticity gauges the response of factor proportions to changes in relative prices.

In summary, the substitution elasticities of male and female labor will

in general be different. In order, for example, to assess the impact of technical change on the supply and demand for male and female labor, the labor inputs must be distinguished by sex, and functional forms must be used that are more flexible than the CD or CES.

(3) Farmer Efficiency

Fixed effects methods are used to estimate the production functions, following the production frontier literature. Fixed effects allows control of unobserved or unobservable, and hence omitted, farm-specific factors.⁴ The production function is to be interpreted as the maximum possible output attainable using the best practices. The fixed effect for the farmer using the best practices is set at zero, so the fixed effects must be at most zero.⁵

A simple model where varying intercepts capture the differences in farm practices (and thus efficiency) is specified (Forsund et al., 1980).⁶

$$y_{kl} = \alpha + v'_{kl} \beta + e_{kl} - u_k \quad (6)$$

where k indexes farm and l indexes plot. e_{kl} is the symmetric random component of error, representing the factors beyond the control of the farmers such as weather and pestilence. u_k represents the factors conceptually under the farmer's control, such as intelligence, risk aversion, and other factors that would in general be correlated with the level of other inputs, v_{kl} . Thus u_k in some sense represents technical "inefficiency" and must be non-negative.

u_k are identically and independently distributed, with mean μ and variance σ_u^2 , and are independent of the e_{kl} . If we write

$$\alpha^* = \alpha - \mu \quad (7)$$

$$u_k^* = u_k - \mu$$

then we can write

$$y_{kl} = \alpha^* + v'_{kl} \beta + e_{kl} - u_k^* \quad (8)$$

Now the error term has zero mean, and most of the results of panel data literature can be applied directly (except those that depend on normality).

$$\text{Let } \alpha_k = \alpha - u_k = \alpha^* - u_k^* \text{ so}$$

the equation estimated can be written

$$y_{kl} = \alpha_k + v'_{kl} \beta + e_{kl} \quad (9)$$

The estimators α_k and β will be unbiased if the farm fixed effect, u_k , is uncorrelated with the regressors. In general, the estimators will be biased. (Even if unbiased, strong conditions are required for consistency.)

The preferred estimator will be the dummy variable or the within estimator. Apply OLS to (9) after expressing all data in terms of deviations from the farm-household mean. The farm-specific intercepts (that include the fixed effects) can be recovered as the farm means of the residuals. The virtues of the within estimators include unbiasedness. Further, consistency does not depend on the distribution of the effects -- the model treats them as fixed, and the estimators are consistent as long as either k or ℓ (number of farms or plots) becomes large.

The disadvantages of the within estimators include the impossibility of

including regressors that are invariant with respect to households, such as farmer's age, schooling, composition of family (which proxy intelligence, risk aversion, and other unobservable factors), so the interpretation of the u_k 's as a measure of "inefficiency" may be a bit tenuous.⁷ But since my interest is in the marginal productivities and the substitution elasticities between factors, all functions of the estimated parameters, I used fixed effects methods to insure consistent and unbiased parameters.

Choice of estimation methods hinges on the statistical significance of the farm specific fixed effects. The significance is tested by comparing the unrestricted with the restricted estimators. The unrestricted estimators come from the regression with the fixed effects transformation. Basically, the fixed effects are not restricted to zero. The restricted estimator comes from the regression without the fixed effects, which corresponds to conventional OLS estimation of the production function.

II. Data

Parameters of the generalized quadratic form of the production function are estimated for each of three major crops grown in the villages from which the data were collected. The survey was conducted by the International Crops Research Institute in the Semi-Arid Tropics over the years 1975 through 1978. The six villages in the original sample were selected as representative of the broad agroclimatic conditions of south central arid India.⁸

Certain social and economic features were incorporated into the model, namely, the division of labor in agricultural production between men, women, and children (Table 1 lists hours worked by men, women, and children in

Table 1

Relative Importance of the Labor of Men, Women, and Children
in Six Villages in Semi-Arid Tropical India
(In Number of Hours; Percentage of Total Hours in Parentheses)

| Task ^a | Men | | Women | | Children | |
|-------------------|---------|---------|---------|---------|----------|--------|
| Ia. | 62,676 | (72.7%) | 23,265 | (27.0%) | 234 | (.3%) |
| Ib. | 21,719 | (15.6%) | 117,105 | (84.1%) | 391 | (.3%) |
| II. | 52,933 | (89.4%) | 5,590 | (9.4%) | 716 | (1.2%) |
| III. | 54,413 | (32.5%) | 110,818 | (66.1%) | 2369 | (1.4%) |
| IV. | 712 | (81.1%) | 165 | (18.8%) | 0 | (0%) |
| All | 192,452 | (42.5%) | 256,943 | (56.7%) | 3710 | (.8%) |

Notes:

^aCategory Ia. includes field preparation, manuring and fertilizing land, and minor and annual repairs to bunds, fences, etc.

Category Ib includes sowing, resowing, transplanting or planting, and weeding and thinning.

Category II includes interculturing, irrigation, plant protection and watching.

Category III includes harvesting and harvest processing.

Category IV includes supervision and management (X), and is excluded from further consideration in this paper.

agricultural production by task); the range of agricultural methods used by the farmers, from modern to traditional; and the Leontief nature of the harvesting and post-harvesting stage production function.

The three major crops grown are sorghum (a grain), rice, and legumes. The inputs included are labor of men, labor of women, and labor of animals -- all in hours expended in tasks comprising the so-called first stage of production. Land plot size is included as well. Hours that the irrigation machines (oil and electric pumps) are included in the regression for rice. (See Table 2 for summary statistics.)

Fertilizer and pesticides were excluded because these were used by only one farmer. Organic manure is captured in the hours spent in collection and application, hours included under labor input. Variation in land quality -- due to variation in soil type, or drainage facilities -- is relegated to the error term, as each observation is a plot sown in the crop. Variation in human and animal labor quality is also omitted, as are differences in quality of physical farm capital. The input specification is problematic but tractable.⁹

III. Results

Two results stand out in Tables 3 and 4. The farm-specific effects were significant (at the 1% level) in the production of sorghum, but not in the production of rice or legumes reflecting a wider range of technologies or resource quality in sorghum production. Second, male and female labor were found to be separable in the production of legumes, but not in the production of sorghum and rice, indicating different substitution elasticities between

Table 2
Regression Sample Statistics

| | Sorghum | | Legumes | | Rice | |
|--------------------------------|---------|--------------------|---------|--------------------|----------|--------------------|
| | Mean | Standard Deviation | Mean | Standard Deviation | Mean | Standard Deviation |
| Output (kgms) | 249.71 | 349.48 | 94.810 | 170.865 | 1420.187 | 1079.65 |
| Preharvest Inputs ^a | | | | | | |
| Female (hours) | 54.89 | 119.83 | 16.84 | 53.90 | 443.35 | 304.59 |
| Hired | 42.55 | 111.89 | 11.03 | 43.85 | 404.77 | 296.57 |
| Family | 12.39 | 33.47 | 5.80 | 22.74 | 38.58 | 44.81 |
| Male (hours) | 105.91 | 119.68 | 42.96 | 71.62 | 332.89 | 192.57 |
| Hired | 49.65 | 87.49 | 20.57 | 61.80 | 67.70 | 139.45 |
| Family | 56.29 | 80.92 | 22.39 | 31.02 | 265.28 | 161.74 |
| Bullock (hours) | 69.77 | 74.50 | 28.86 | 45.27 | 110.90 | 114.17 |
| Machine (hours) | 0 | 0 | 0 | 0 | 137.14 | 141.93 |
| Land (hectares) | 2.78 | 2.29 | 1.60 | 1.59 | 1.34 | .87 |
| Harvest Inputs ^b | | | | | | |
| Female (hours) | 81.02 | 118.23 | 46.44 | 77.50 | 215.29 | 153.97 |
| Male (hours) | 56.11 | 67.14 | 22.55 | 33.24 | 87.55 | 64.82 |
| Bullock (hours) | 5.31 | 12.39 | 1.33 | 10.88 | 24.60 | 23.24 |
| Machine (hours) | .025 | .21 | 0 | 0 | 0 | 0 |
| No. Obs. per village | | | | | | |
| Aurepalle | 61 | | 4 | | 56 | |
| Dakur | 13 | | 0 | | 126 | |
| Shirapur | 97 | | 122 | | 0 | |
| Kalman | 263 | | 163 | | 0 | |
| Kanzara | 53 | | 31 | | 0 | |
| Kinkheda | 77 | | 28 | | 0 | |
| No. Obs. 1975-1976 | 260 | | 156 | | 56 | |
| No. Obs. 1976-1977 | 304 | | 92 | | 126 | |
| No. Obs. | 564 | | 348 | | 182 | |

^aInputs included in the regression.

^bInputs excluded from the regression.

Table 3

Marginal Products of Factors^a
(F-statistics in parentheses)

| | Sorghum | | Legumes | | Rice | |
|-----------------------|-------------------|-------------------|-------------------|-------------------------------|-------------------|--------------------|
| | (1)* | (2) | (3) | (4)* | (5) | (6)* |
| Female Labor | .838 (13.21) | .206 (.91) | .381 (2.79) | .785 (9.919) | 1.612 (18.66) | 1.525 (19.37) |
| Male Labor | 1.554 (14.36) | .997 (6.94) | 1.127 (5.62) | 1.019 (4.113) | 2.782 (7.53) | 3.135 (9.04) |
| Bullock | -2.418 (13.35) | -1.799 (4.16) | -.712 (1.07) | -.171 ^a (.0557) | -3.073 (3.24) | -2.542 (2.61) |
| Machines ^b | . | . | . | . | -1.126 (1.37) | -.669 (.51) |
| Land | 42.023 (18.26) | 46.226 (20.22) | 32.048 (16.99) | 198.78 (5.214) | 559.59 (12.71) | 519.385 (21.89) |
| No. Obs. | 564 | | 348 | | 182 | |
| F 5% (1%) | 3.84 (6.63) | | 3.84 (6.63) | | 3.84 (6.63) | |

Notes:

^aStarred columns contain marginal products calculated from consistent estimators (footnote 10).

^bMachines are primarily irrigation pumps. Sorghum and legumes are unirrigated, while rice is irrigated.

Table 4
Elasticities of Factor Substitution^a

| Elasticity Between Inputs i and j | Sorghum | | Legumes | | Rice | |
|---|---------|--------|---------|--------|--------|--------|
| | (1)* | (2) | (3) | (4)* | (5) | (6)* |
| Male & Female | -10.67 | 3.785 | 2.075 | .988 | -.772 | -1.021 |
| Male & Animal | .738 | -1.298 | -6.294 | -5.246 | .587 | .533 |
| Male & Land | .488 | -.112 | .402 | .533 | .121 | .00147 |
| Male & Machines | . | . | . | . | -1.321 | -1.143 |
| Female & Animal | .346 | .147 | -6.500 | -3.557 | -.117 | -.142 |
| Female & Land | .312 | .209 | .492 | -.0460 | -.0277 | -.0735 |
| Female & Machines | . | . | . | . | .538 | .545 |
| Animals & Land | -.157 | -.252 | -4.665 | -4.861 | -.370 | -.2733 |
| Animals & Machines | . | . | . | . | -.179 | -.126 |
| Land & Machines | . | . | . | . | .114 | .0219 |

Notes:

^aStarred columns contain elasticities calculated from the consistent estimator.

male and female labor and all other factors in sorghum and rice.

(1) The within estimator then is preferred, because it is unbiased and consistent, in sorghum production (column 1 in Table 3 lists marginal products calculated from the preferred estimator, and column 1 in Table 4 lists substitution elasticities).¹⁰ The OLS estimator is preferred in legumes and rice (columns 4 and 6 in Tables 3 and 4).

The marginal product of female labor in sorghum is significantly different from zero only when the farm effects are included. In fact, the marginal products of both male and female labor are greater, and that of land less, when calculated from the within estimator. This is a result that can be illustrated (following Timmer, 1970) with a simple graph.

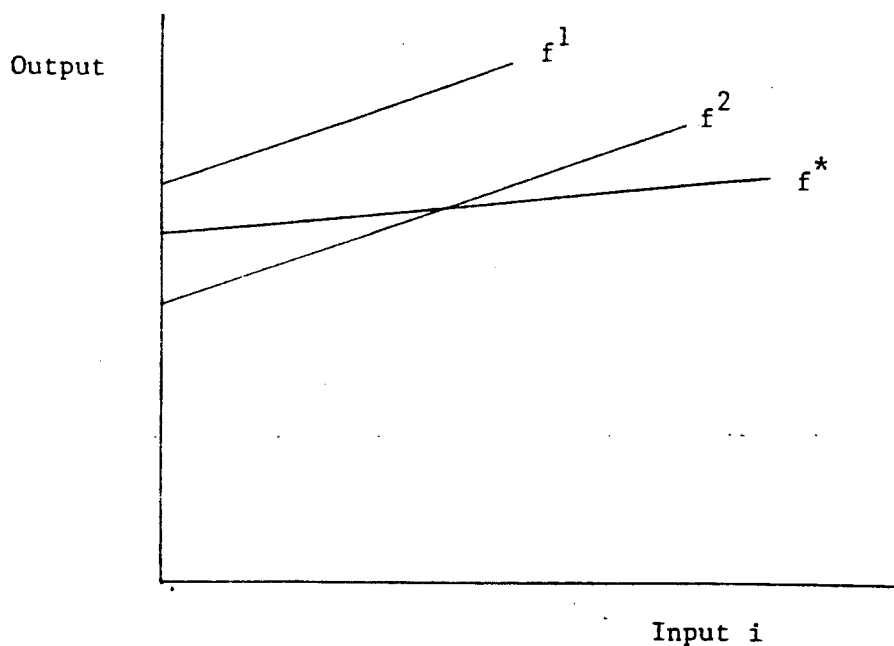


Figure 1

Farmer Efficiency

Suppose there are only two kinds of farmers, call them "efficient" and "less efficient" farmers -- the reality is a continuum of "less efficient" farmers. Their production practices are summarized by the lines f_1 and f_2 . The so-called efficient farmers produce more output with a given input than the less efficient. Differences in "efficiency" were modeled so as to only affect the farm intercept rather than the slope of the production function. Unless the differences in farmer efficiency were incorporated into the model, the slope of the relationship between output and input 1 (the marginal product of input 1) would appear to be f^* . The true marginal product would be greater (as in the case of labor of females).

The source of these farm effects may be resource quality.¹¹ The effects were found to be negatively correlated with the farm average level of inputs per hectare in sorghum. Such a result would be consistent with the suggestion of omitted measures of resource quality (soil type, drainage, quality of other labor and capital inputs). In fact, the finding mirrors the findings reported in a technical paper about sorghum production in the sampled villages (Walker and Rao, 1982). Greater soil quality, measured by an index of salinity, significantly enhanced production of sorghum.

Why would resource quality be important only in the production of sorghum? The significance of the fixed effects in sorghum contrast with insignificance in legumes and rices. I would suggest that this result reflects the range of techniques used by the farmers in each of the crops. The techniques used to grow rice and legumes may, in fact, be more homogenous than those used to grow sorghum.

The production function for high yielding variety rice only was estimated. Technologies, in the specific sense of the seed/fertilizer/irrigation package, would therefore be more homogenous across farms growing rice.

Legumes are primarily grown for home consumption, and in fact, legumes as a category includes a mixture of different kinds of beans. Legumes are intercropped with other vegetables and grains, but only the plots on which legumes were the main crop were included in the regression sample. Legumes are generally of the local unirrigated (rainfed) variety. Like rice, the technologies used may not vary much across farmers.

Sorghum, on the other hand, is grown for both home and market consumption. Technologies used, again, in the specific sense of types of farm tools, seeds, fertilizer, and so on, may be more heterogeneous across farmers. This heterogeneity perhaps is being captured in the farm fixed effects. Use of modern technologies perhaps is being distinguished from use of traditional technologies, and modern technologies are the "best practices" modeled.

(2) The titular results are the magnitude and significance of the marginal product of the labor of women in the production of all three crops, and the wide range of substitution elasticities.¹²

The "unpaid" family laborers contribute importantly to the farm income. Further, the ratio of the marginal products of female and male labor, .486 in rice, .536 in sorghum, and .770 in legumes, lies within the range of ratios of observed wages paid in the six villages, .4 to .78 (Table 5 lists the average of the wages paid). Farmers appear to be rational in that wages are in line with marginal products.

Different marginal products do not imply different substitution elasticities between the labor of men and women, on the one hand, and all other factors.¹³ Likelihood ratio tests however indicate that male and female labor are not (weakly) separable in the production of two of the three crops -- sorghum and rice, results reflected in the substitution elasticities

Table 5

Average Observed Wages in the Six Villages
1975-1976 and 1976-1977^a
(number of observations in parentheses)

| Village | Male | Female | Bullock Pair | Male | Female | Bullock Pair |
|-----------|-------------|-----------|-----------------|-----------|-----------|-----------------|
| | rupees/hour | | | | | |
| Aurepalle | .33 (8) | .24 (109) | .72 | .31 (13) | .21 (43) | .94 |
| Dokur | .39 (95) | .27 (414) | .97 | .36 (48) | .28 (171) | 1.20 |
| Shirapur | .35 (55) | .14 (55) | 1.24 | .42 (22) | .18 (47) | 1.54 |
| Kalman | .35 (43) | .19 (65) | 1.53 | .38 (139) | .24 (206) | 1.10 |
| Kanzara | .52 (110) | .25 (132) | 1.28 | .48 (139) | .24 (206) | 1.10 |
| Kinkhada | .39 (129) | .17 (187) | 1.15 | .47 (107) | .23 (154) | 1.36 |

Notes:

^aCalculated from statistics presented in Asokan (1980).

(columns 1, 4 and 6 in Table 4).

Male and female labor are good complements in sorghum, but male labor is more substitutable for animal power and land, although the magnitude of the elasticity is less than one. Male and female labor are combined in relatively fixed proportions with animal power and land. Similarly, male and female labor are complements in rice production, and again male labor is more substitutable with land and animal power than female labor. Male labor is complementary with machines (mainly irrigation pumps), while female labor is substitutable. In contrast, male and female labor are found to be good substitutes in legumes, and equally complementary with animal power. Both male and female labor are combined in relatively fixed proportions with land.

IV. Discussion and Dynamic Extensions

Intuitively, male and female labor should be, on the margin, substitutable. Male and female labor per se are not complementary, rather male and female must be proxying some attribute of the laborers or the tasks performed.¹⁴

Certain tasks are observed to be performed mainly by men -- field preparation, fertilization and manuring, sowing, interculturing and irrigation. Women perform mainly weeding and thinning tasks, nursery bed raising, transplanting and planting. Harvesting, threshing and harvest processing are shared by men and women. (Table 2 describes the allocation of time to agricultural production by gender and task.)

Substitutability or lack of substitutability between tasks is perhaps being captured in the range of substitution elasticities. Hours ploughing

with bullocks would be substitutable for hours manuring and sowing, tasks mainly performed by men, rather than bullocks and men. Ploughing may be less substitutable with nursery rice bed raising, tasks mainly performed by women, rather than bullocks and women, etc.

In other words, the results suggest misspecification. A dynamic model that distinguishes labor input by task as well as by gender is more appropriate than the static model used. Agricultural production has a time and spatial dimension that is not important in industrial production. Land preparation precedes sowing, and weeding and thinning, and as well must take place according to the seasons and weather -- harvesting must be done before the monsoons. At any "point" between land preparation and harvesting, the crop may be partially or totally destroyed by drought, or flood, or other unlooked for catastrophe. A more dynamic model would distinguish the stages of production: the land is prepared, the seeds are sown in the prepared land, the young plants are irrigated, and then thinned and weeded, and watched. The crop left standing is harvested.

In symbols,

$$y_1 = f_1(V_1, e_1)$$

$$y_2 = f_2(y_1, V_2; e_2)$$

.

.

$$y_n = f_n(y_{n-1}, V_n; e_n),$$

where V_i is the vector of labor and non-labor inputs in stage $i = 1, \dots, n$, and y_i is the output of stage i . y_n would be harvested and processed labor, and y_1 would be prepared land.

Preparatory tillage, sowing, weeding and thinning, and harvesting, are not in general freely substitutable in the production process. For example, when furrows are more deeply ploughed, the seeds may be more densely sown because the roots penetrate deeper. Weeding requirements are diminished as a consequence. Hours spent in ploughing and sowing would be complementary, but both would be substitutes for hours spent in weeding.

If input data were recorded by stage, and the stage outputs measured (or measurable), then the triangular system could be estimated. When inputs are included in aggregate (over all stages), consistent and unbiased estimation turns on several strong assumptions about the error structure that are valid only if the farmers choose the input allocation plan at the onset, and never deviate from the plan.¹⁵

The categories -- male and female labor -- would be capturing some of the substitution relationships between different tasks because of the strict division of labor. Male and female labor could be complementary in sorghum and rice, but substitutable in legumes because of the different natures of the tasks required to grow the different crops.

The magnitude of the substitution elasticities suggests that the CES and the CD would be inappropriate. Of course, the standard errors of the elasticities should be calculated to determine whether the elasticities are significantly different from each other, or one. The availability of the panel data of the sort used in this paper should redirect attention to agricultural production functions. How valid are the simplifying assumptions? How does one interpret the economic statistics calculated from the parameters of aggregate production functions? Indeed, it may be the substitution possibilities between production operations, or that between crops and seasons, rather than between factors, that are of theoretical and applied interest.

V. Summary

Flexible functional form of production functions for each of three crops grown in the Indian village sample were estimated using fixed effects methods. Production functions, rather than the derived first-order conditions (input demand functions) or the dual cost functions, accommodated the panel nature of the data, as well as the limitation of lack of variation in prices. Production was simply modeled as a two stage process that corresponded to the observations of agriculturalists, to wit, the harvesting stage was a Leontief-type process. The two stages were maintained to be separable for purposes of econometric tractability. Fixed effects methods were used to control for unobserved omitted farm-specific factors, all collected under the rubric of "farm efficiency."

The marginal product of female labor -- both family and hired -- was found to be significant in the production of three major crops grown in the six sample villages. Further, female and male labor were found to be asymmetrically substitutable with respect to all other factors of production, the asymmetry perhaps arising from misspecification. A more dynamic model of agricultural production enumerating the sequence of production operations may be a more appropriate model, and should be the object of further research.

FOOTNOTES

¹Statistical weak separability (WS) is indicated here, i.e. if the partition of inputs is WS, then the marginal rates of substitution between inputs within the partition are independent of the level of all inputs outside the partition. It is obvious that the maintained separability of the two stages may be tested.

²Denny and Fuss (1977) have shown that the CES and the Cobb-Douglas production functions are nested within the flexible functional translog form. The generalized translog with partially strong separability of inputs imposed is the CES, and further, with strong separability of inputs, reduces to the Cobb-Douglas.

³Denny and Fuss (1977) have derived the necessary parameter restrictions on the generalized translog.

⁴When the production function is estimated directly, there arise two potential sources of systematic error that should be considered, namely, the farm-specific effect and the errors in measurement of the inputs.

⁵Marschak and Andrews (1944) introduced the firm-specific effect. Hoch (1958, 1962), Mundlak (1961, 1962), and Mundlak and Hoch (1965) cited in Fuss, et al. (1978), proposed methods for controlling for the effect, so that the production function estimates would be consistent estimates. Hoch (1962), for example, deals with panel data, and constructs an index of farm "efficiency." Aigner et al. (1977) and Meeusen and van den Broek (1977) bridged the farm-specific effect and the production frontier literature, proposing that the one-sided error term of the production frontier consists of a one-sided farm-specific and a symmetric random effect.

⁶The economists at ICRISAT reported that differences in the abilities of farmers, in their opinion, considerably affected yields.

⁷Hausman and Taylor (1981) showed that if it could be maintained that some of the effects were uncorrelated with some (but not all) of the regressors, then the farm invariant effects could be recovered. Their method is a hybrid of generalized least squares (GLS) and the within methods. Consistency and unbiasedness of the GLS estimator depends on the condition that the farm effects are uncorrelated with all regressors, so GLS would not be appropriate.

⁸Jodha et al. (1977) and Binswanger and Jodha (1978) summarize the village survey methods. Binswanger et al. (1980) describe the village labor markets, and Ryan and Ghodake (1980) present statistics of labor market participation by sex, season, etc.

⁹Observations are stacked, and fixed effects methods are used. Season dummies were included in the list of regressors. There are two seasons in each agricultural year when the crops may be grown, namely, the rainy season (Hindi: kharif) and the post-rainy season (Hindi: rabi). Yields are expected to vary across seasons due to moisture and temperature differences. Moreover, soil moisture is known at planting time for rabi crops, and as well, the incidence of disease and pests is less in the rabi season.

¹⁰The marginal products (MP) of all included factors are calculated from the parameter estimates and evaluated at the mean of the factor level,

$$mp_i = \beta_i + \sum_j \beta_{ij} \bar{v}_j$$

The MP is the change in kilograms output for an hour's change in input level (or hectares land) from the average.

Substitution elasticities (Allen-Uzawa) are calculated as well from the estimates and evaluated at the average factor level,

$$s_{ij} = \frac{(m_{pi})(\bar{v}_i) |D_{ij}|}{(\bar{v}_i)(\bar{v}_j) |D|}$$

where s_{ij} is the elasticity of substitution between factors i and j , D_{ij} is the ij th cofactor of the border Hessian,

$$D = \begin{bmatrix} \frac{\delta^2 F}{\delta v_i \delta v_j} & mp_1 \\ & \vdots \\ mp_1 \dots mp_n & mp_n \\ & 0^n \end{bmatrix}$$

and mp_i is the MP of factor i . F is the production function, $y = F(v_i)$.

The MP value calculated is the exact value, not the first partial derivative of the true production function, the interpretation given to β_{ij} when the generalized quadratic is, in turn, interpreted as an approximation to the true function rather than the true function.

¹¹A survey of production function studies does not indicate a consensus on either the significance nor the sources of the farm-specific effects. Timmer (1970) analyzed a time series of US state level farm production data, and found the effect to be associated with the level of inputs. Kalirajan (1981) estimated Cobb-Douglas production functions, using cross-sectional data from seventy farmers in Tamil Nadu State, India, growing post-rainy season rice. He found an association with the farmer's experience and the number of times the extension worker visited the farm. In contrast, Khandker, et al. (1983) estimated net revenue functions combining price and input quantity data across all crops grown by two hundred Bangladeshi households, and concluded that all farmers used the best practices. Obviously, the effects must be empirically determined for particular villages or regions or states.

¹²The standard errors of the elasticities have not yet been calculated.

¹³Different wages are not a sufficient condition for the non-existence of a consistent input aggregate. In fact, a weighted average of male and female labor, with the weights proportional to the observed relative wages, has been used as a labor aggregate. Strict separability of factors will be reflected in the technical substitution elasticities derived from the regression coefficients, not in marginal products.

¹⁴Taylor (1982) presented this hypothesis in the context of US manufacturing.

¹⁵This hypothesis is concisely and rigorously presented by Antle (1982).

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